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PRACTICAL ASPECTS OF IMPLEMENTATION OF THE PARALLEL ALGORITHM FOR SOLVING PROBLEM OF CTENOPHORE POPULATION INTERACTION IN THE AZOV SEA*

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The paper covers the development and researching mathematical model of interaction processes between plankton and ctenophore populations based on the modern information technologies and computational methods, which leads to increase of the accuracy of predictive modeling of the ecology situation in shallow water in summer. The model takes into account the following: the transport of water environment; microturbulent diffusion; nonlinear interaction of plankton and ctenophore populations; biogenic, temperature and oxygen regimes; influence of salinity. The computational accuracy is significantly increased, and computational time is decreased at using the calculation method based on partially filled cells for discretization of model. The practical significance is the software implementation of the proposed model, the limits and prospects of its practical use are defined. Experimental software was developed based on multiprocessor computer system, which is intended for mathematical modeling of possible progress scenarios in shallow waters ecosystems on the example of the Azov Sea in summer. We used decomposition methods of grid domains in parallel implementation for computationally laborious convection-diffusion problems, taking into account the architecture and parameters of multiprocessor computer system.

Keywords: mathematical model, hydrological processes, expedition research, ctenophore, Azov Sea, parallel algorithm, multiprocessor computer system.

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Introduction

The biological contamination is one of the serious global problems of aquatic ecology. The invasion of aquatic species often occurs at discharging the ballast waters of ships. Mathematical modeling of invasions is the great theoretical interest problem with practical relevance, and refers to the actual and prospective direction of aquatic ecology. One of development stages of a complete model of the Azov-black Sea ecosystem is the development and research models of hydrological processes, including models of interaction between phyto- and zooplankton.

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Complex models of interacting populations of key aquatic organisms contains: different species of phytoplankton, zooplankton and jelly macroplankton, including ctenophores populations *Mnemiopsis leidyi* and *Beroe ovata* predators.

It's known that medusa *Aurelia aurita* was the dominant species in jelly plankton of the Black Sea until the mid-80s. During the prevalence of *Aurelia aurita* ecosystem of the Black Sea was in relative balance, and the fish catch remained at approximately constant level [1].

In 1982 in the Black sea the *Mnemiopsis leidyi* appeared which eat the zooplankton, the eggs and larvae of fish and shellfish. It leads to the decreasing the amount of food for fish by 30 times in comparison with the period of 1978 – 1988. It was one of causes of "anchovy crisis". The number of individuals in 1 m^3 reached 400 units.

Mnemiopsis leidyi populated the Caspian Sea through the Volga-Don Channel in 1999. As a result, 75 % of zooplankton were dead, that had a great influence on the food chains of the sea. *Mnemiopsis leidyi* was first spotted in the North and Baltic Seas. The predatory *Beroe ovata* eating only ctenophora *Lobata* appeared in 2006.

Each period of ctenophore appearing in great amount is characterized by changes at the structure of plankton population and leads to the fish reproduction [1, 2].

The paper covers the development and researching mathematical model of interaction processes between plankton and ctenophore populations based on the modern information technologies and computational methods, which is lead to increase the accuracy of predictive modeling the ecology situation in shallow water in summer. The model takes into account the follows: the transport of water environment; microturbulent diffusion; nonlinear interaction of plankton and ctenophore populations; biogenic, temperature and oxygen regimes; influence of salinity. The computational accuracy is significantly increased, and computational time is decreased at using the calculation method based on partially filled cells for discretization of model. Experimental software was developed based on multiprocessor computer system, which are intended for mathematical modeling of possible progress scenarios in shallow waters ecosystems on the example of the Azov Sea in summer. We used decomposition methods of grid domains in parallel implementation for computationally laborious convection-diffusion problems, taking into account the architecture and parameters of multiprocessor computer system.

The first section of the paper is devoted to the expedition researches of the Azov Sea, which are used for calibration and verification of the developed model of plankton and ctenophore interaction. The second section of the paper is devoted to description of the developed mathematical model of biological kinetics. The construction and research a discrete analog of the developed model using the schemes of high order of accurasy and partially filled cells are described in the third section. Solution method of grid equations, obtained during the discretization of the developed model of biological kinetics, that are described the plankton and ctenophore interaction in the Azov Sea, are described in the fourth section. A parallel variant of method for solution grid equations is described in the fifth section. The sixth section is devoted to the description of software complex. Results of numerical experiments of the software complex, numerically implemented the proposed model problem are given in the seventh section. The main results and future research directions are given in conclusion.

1. Expedition researches of the Azov Sea

The Institute of Oceanology by P.P. Shirshov has conducted longtime researches of ecosystems in different regions of the Azov-Black Sea basin, the results of which allowed to identify the influence of jelly macroplankton in the functioning of its ecosystems [2]. A comparison of the number of macro- and meso-zooplankton obtained using different equipment and techniques (nets, bottles, underwater video survey and direct counting with underwater installations), and the analysis of catch of different plankton nets used during the expeditions, showned that M. Leidyi invasion caused a significant decrease in food stocks of zooplankton and plankton-feeding fish [1]. The intensive absorption of bivalve larvae by the M. leidyi ctenophore was found in the coastal waters of the North-Eastern part of the Black Sea. The seasonal population dynamics of ctenophores, their interaction and age structure, basic physiological and ecological characteristics were researched, and the weight of dry samples, the carbon content, the ratio of length/mass and quantitative parameters of ctenophores during feeding and reproduction were measured in laboratory experiments. Daily analysis of the *M. Leidyi* population dynamics [1] revealed a periodic wave of the reproduction intensity every 10-15 days, and a gradual increasing was at maximum in August. Further population growth is interrupted at the end of August by a sharp increase in the *B. Ovata*, which led to the almost complete disappearance of the *M. Leidyi* in November-December.

The main problem of expedition researches is the complex researching of modern conditions and spatial-dimension measurements of the hydrobiological, hydrological and hydrochemical regimes of the Azov Sea and Taganrog Bay. The route on the research vessel "Deneb" was made in July 2017 by the staffers of the Don State Technical University, Southern Scientific Center of RAS at the Azov Sea basin. More than 20 complex oceanographic stations were researched, and samples of water, plankton and benthos were obtained. And shipboard observations of birds and marine mammals were made. Sampling of phytoplankton was carried out by the sampler in several horizons. The number and biomass of organisms of each taxonomic group was calculated for 1 m^3 of water. Zooplankton samples were taken by filtration through plankton net of Epstein or Jedi [3].

The tables of standard scales of organisms, composed of Mordecai-Boltovskii, were used for calculation of the biomass of zooplankton. The number and biomass of organisms of each taxonomic group was calculated for 1 m^3 of water.

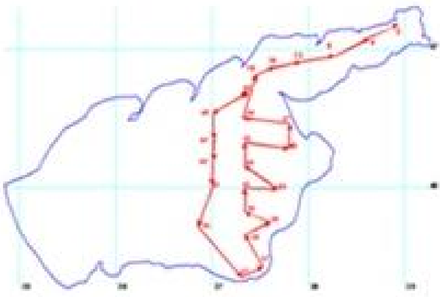
Samples of jelly plankton were taken at all stations, a few vertical climbs by the ctenophora net without the extension of the cone and one oblique harvest Bongo net were made. The biomass of the *Mnemiopsis leidyi* and *Beroe* ctenophores and their sizes was calculated in each sample by volumetric method.

Sampling of zoobenthos was carried out with a Petersen grab or the benthic frame with a square grip of 0,025 m^2. The number and biomass of organisms of each taxonomic group was calculated for 1 m^2 of water.

The hydrochemical conditions were researched (the oxygen content, pH, salt and nutrient composition of water). Sampling of water was carried out by the samplers. Processing of samples was carried out according to the conventional methods of hydrochemical studies in conditions equipped with necessary laboratory equipment (Fig. 1).

2. Problem statement

Specific features of survival were taken into account at developing spatially-inhomogeneous models of interacting populations of main aquatic organisms, including populations of phytoplankton, zooplankton, and ctenophores *Mnemiopsis leidyi (M. leidyi*) and *Beroe ovata* predators. It is known that *M. leidyi* is a species with simple short life cycle and very high fertility,



a) The route of the vessel

b) SRV "Deneb"

c) Hydrochemical researches

d) Biological researches

Fig. 1. Expeditionary researches in 2017

its population is increasing rapidly, exponentially. "The correlation between the linear dimensions of the ctenophore and the consumption rate of zooplankton" is characterized for *M. leidyi.* This species survives in unfavorable conditions of new water with a dedicated adaptation mechanism. These jelly fish feed on benthic invertebrates, eggs and larvae of fish. The youngest age group of the ctenophore feed on microplankton: protozoa, phytoplankton, doubleline (*Nauplius*, from ancient Greek. nauplios – "a floating animal with a shell") stages of copepods (Ref. *Copepoda*) [4]. The increase of *M. leidyi* occur at the expense of pedogenesis (the larvae develop unfertilized eggs, which give the start to a new generation).

For modeling dynamics of forage populations of ctenophores was used in situ and evaluation of the distribution of the nutrient status of the entire area of the range of ctenophores with the zoning of the Azov Sea were taken into account the main ecological and biological processes: growth, mortality, metabolism (estimation of influence of waste products of *M. leidyi* (mucus, pellets) and its decomposition products on the abundance of microflora and the formation of proteolytic and amylase activity), the physical transfer of aquatic organisms, including active and passive migration, and so account for the movement of water flow, diffusion, taxis.

The last month of summer is the period of active development of the zooplankton, but under favorable food and temperature conditions, the rate of development of *M. leidyi* is much higher. In the process of the conducted researches it was established that the indices of biodiversity of phytoplankton are higher in places of mass congestion of *M. leidyi.* As a result of intensive development of phytoplankton-based indicators of chlorophyll "a" is also significantly higher in habitats of *M. leidyi*, which is clearly seen on cards of direct measurements of chlorophyll and SeaWiF satellite data. Primary product selection of *M. leidyi* is the ammonium nitrogen [5].

The ratio of organic nitrogen to mineral form in the habitat area of *M. leidyi* increases. Waste products of the *M. leidyi* also is mucus constantly reset it with the surface of the body, which leads to an increase of suspended organic matter in the water, and as a result creates favorable conditions for the development of microheterotrophs, primarily bacteria, and contributes to a significant increase in their numbers.

An active regeneration of inorganic forms of biogenic elements (nitrogen, phosphorus, silicon) occurs as a result of the bacteria activity. Extracted nutrients are used by phytoplankton, increasing its biomass. In terms of Azov areal, metabolic rates of *M. leidyi* estimated from oxygen consumption and release of ammonium is approximately 1.5 times higher than the oceanic forms of the protein consumed zooplankton assimilation (growth), ctenophore uses only about 26 % nitrogen, 20 % is dissipated in the environment in the form of low molecular weight peptides and amino acids and 54 % of nitrogen dissimilitude to ammonium. In marine ecosystems, the *M. leidyi* acts as a "short dimension" of the food chain, intercepting and mineralize flows of organic matter zooplankton, regenerate nutrients and thus stimulate the development of phytoplankton. After the invasion of the ctenophore in the Azov sea doubled primary production with a concomitant increase in the content of organic substances in water and sediments and its turnover. One of the consequences of changes in trophic chains in the ecosystem of the Azov Sea in the result of introduction of the *M. leidyi* is increasing of the proportion detatoko link [5].

M. leidyi can indirectly regulate the dynamics and distribution of summer phytoplankton through consumption of zooplankters phytophages, producing the effect of "cascade" to lower trophic levels through zooplankton on the phytoplankton and chlorophyll "a". Grazing by *M. leidyi leidyi* of meroplankton (larvae of benthic animals) and demersal plankton leads to a decrease in biomass of the representatives of the benthos.

The spatial distribution of salinity and temperature were taken into account at modeling of hydrological processes (the plankton and ctenophores interaction) in a shallow water – the Azov Sea. The number of *M. leidyi* depends on the distance of its penetration, and is determined by the direction and speed of wind currents, contributing to the drift of water with high salinity. The *M. leidyi* can live and multiply only when salinity is above 4,3 ‰ (for the Caspian Sea) [7]. The distribution of *M. leidyi* in the Azov Sea is limited to isohalines 3 ‰ [8] and occured due to its penetration of the Black Sea, where every year it re-colonized in the spring or early summer, and living until October, then dead at the decreasing of the temperature below 4 °C.

The field measurements obtained at the expeditions at the Azov-Black Sea basin and the "Analytical GIS" portal data developed by the Institute for Information Transmission Problems of the Russian Academy of Science (IITP RAS) at modelling in this paper [6].

The model for description the ecological and biological interaction process between ctenophores and plankton has the form:

$$(P_i)'_t + \frac{1}{2}\sum_{\alpha=1}^{3}\left\{U_\alpha (P_i)'_{x_\alpha} + (U_\alpha P_i)'_{x_\alpha}\right\} = \mu_i \Delta P_i + \frac{\partial}{\partial x_3}\left(\nu_i \frac{\partial P_i}{\partial z}\right) + \psi_i,\ i \in \overline{1,9}. \quad (1)$$

$$\psi_1 (P_1, P_2, P_3) = \{\alpha_1 P_3 - \delta_1 P_2 - \varepsilon_1\} P_1, \psi_2 (P_1, P_2) = \{\alpha_2 P_1 - \varepsilon_2\} P_2,$$

$$\psi_3 (P_1, P_3, P_4) = \{\alpha_3 P_4 - \delta_3 P_1 - \varepsilon_3\} P_3, \psi_4 (P_3, P_4, P_5) = \{\alpha_4 P_5 - \delta_4 P - \varepsilon_4\} P_4,$$

$$\alpha_4 = (\alpha_{04} + \gamma_4 M_4)\,\psi_5\,(P_1, P_2, ..., P_9) = \sum_{i=1, i\neq 5}^{9} \varepsilon_i P_i - \delta_5 P_4 P_5 + B\left(\bar{P}_5 - P_5\right) + f,$$

$$\psi_m\,(P_1, P_2, P_3, P_4, P_6, ..., P_9) = \sum_{l=1}^{4} k_l P_l - \varepsilon_m P_m;\ m \in \overline{6,9},$$

where P_i are concentration values,$i \in \overline{1,9}$: 1, 2 are ctenophores *Mnemiopsis leidyi* and *Beroe ovata*; 3 is zooplankton; 4 is phytoplankton; 5 is the biogenic matter; 6, 7, 8, 9 are metabolites of ctenophores (6, 7) and plankton (zoo- (8) and phyto- (9)); ψ_i are functions of trophic interactions; α_l is the function of ctenophores and plankton growth, $l = \overline{1,4}$; α_{04},γ_4 are the growth rate of phytoplankton in the absence of metabolite and the effect parameter; B is the rate of arrival of nutrients P_5; $\bar{P}_5$ is the maximum possible concentration of nutrients; ε_l is a coefficient taking into account mortality of the l-th species; ε_m are the coefficients of metabolite decomposition, $m = \overline{6,9}$; k_l are the excretion coefficients of l-th species (ctenophores ($l = \overline{1,2}$), zooplankton ($l = 3$), phytoplankton ($l = 4$)); δ_1, δ_3, δ_4 are coefficients of wastage due to grazing $f = f\,(x_1, x_2, x_3, t)$ is the source function S (polution); u is the velocity field of water flow; $U = u + u_{0i}, U = (U_1, U_2, U_3)$ is the rate of convective mass transfer; u_{0i} is the deposition rate of the i-th substance; $\mu_i \nu_i$ are the diffusion coefficients in the horizontal and vertical directions of the i-th substance.

The computational domain G is a closed basin, limited by the undisturbed water surface Σ_0, bottom $\Sigma_H = \Sigma_H(x_1, x_2)$ and the cylindrical surface σ for $0 < t \leq T_0$. $\sum = \sum_0 \cup \sum_H \cup \sigma$ – the sectionally smooth boundary of the domain G.

Boundary conditions for the system (1) are following:

$$P_i\ = 0\ on\ \sigma,\ U_n < 0;\ \frac{\partial P_i}{\partial n} = \varphi_i\ \ on\ \sigma,\ U_n \geq 0; \tag{2}$$

$$P'_{i,z}\ \ = 0\ \ on\ \Sigma_0;\ P'_{i,z} = -\beta_i P_i\ \ on\ \Sigma_H,$$

where β_i is the i-th bottom material absorption coefficient.

Add to (1) following initial conditions:

$$P_i|_{t=0} = P_{i0}(x_1, x_2, x_3),\ i = \overline{1,9}. \tag{3}$$

3. Construction of a discrete analog of the model

Each equation of the system (1) – (3) can be represented by the diffusion-convection-reaction equation in the two-dimensional case:

$$c'_t + uc'_x + vc'_y = \left(\mu c'_x\right)'_x + \left(\mu c'_y\right)'_y + f$$

with boundary conditions:

$$c'_n\,(x, y, t) = \alpha_n c + \beta_n,$$

where u, v are water velocity components; μ is the turbulent exchange coefficient; f is the function, describing the intensity and distribution of sources.

The uniform grid was defined for numerical implementation of the discrete mathematical model [7]:

$$w_h = \{t^n = n\tau,\ x_i = ih_x,\ y_j = jh_y;\ \ n = \overline{0, N_t},\ i = \overline{0, N_x},\ j = \overline{0, N_y};$$

$$N_t\tau = T,\ N_x h_x = l_x,\ N_y h_y = l_y\}\,,$$

where τ is the time step; h_x, h_y are spatial steps; N_t is the upper time boundary; N_x, N_y are spatial boundaries.

Discrete analogs of convective uc'_x and diffusive $\left(\mu c'_x\right)'_x$ operators of the second order of accuracy in the case of partially filled cells can be written as:

$$\left(q_0\right)_{i,\,j} uc'_x \simeq \left(q_1\right)_{i,\,j} u_{i+1/2,\,j}\frac{c_{i+1,\,j}-c_{i,\,j}}{2h_x} + \left(q_2\right)_{i,\,j} u_{i-1/2,\,j}\frac{c_{i,\,j}-c_{i-1,\,j}}{2h_x}, \tag{4}$$

$$\left(q_0\right)_{i,\,j} \left(\mu c'_x\right)'_x \simeq \left(q_1\right)_{i,\,j} \mu_{i+1/2,\,j}\frac{c_{i+1,\,j}-c_{i,\,j}}{h_x^2} - \left(q_2\right)_{i,\,j} \mu_{i-1/2,j}\frac{c_{i,\,j}-c_{i-1,\,j}}{h_x^2} - \tag{5}$$

$$-\left|\left(q_1\right)_{i,\,j} - \left(q_2\right)_{i,\,j}\right| \mu_{i,\,j}\frac{\alpha_x c_{i,\,j}+\beta_x}{h_x},$$

where $q_l, l \in \{0,1,2\}$ are coefficients, describing the "fillness" of control domains.
The approximation error of (4) in the case $\left(q_1\right)_{i,\,j} = \left(q_2\right)_{i,\,j} = 1$ has the form [8]:

$$u_{i+1/2,\,j}\frac{c_{i+1,\,j}-c_{i,\,j}}{2h_x} + u_{i-1/2,\,j}\frac{c_{i,\,j}-c_{i-1,\,j}}{2h_x} =$$

$$= u_{i,\,j}\left(c_{i,\,j}\right)' + \frac{\left(c_{i,\,j}\right)'\left(u_{i,\,j}\right)''}{4}h_x^2 + \frac{u_{i,\,j}\left(c_{i,\,j}\right)'''}{6}h_x^2 + \frac{\left(u_{i,\,j}\right)'\left(c_{i,\,j}\right)''}{4}h_x^2 + O\left(h_x^4\right).$$

Therefore, the operator $uc' - c'u''h^2/4 - uc'''h^2/6 - u'c''h^2/4$ must be approximated by a scheme of the second order of accuracy for approximation the convective transport operator uc' by a difference scheme of the fourth order of accuracy.
The approximation of the convective transport operator uc' by a difference scheme of the fourth order of accuracy has the form:

$$\begin{gathered}
\left(q_0\right)_i L\left(c\right) = -\left(q_1\right)_i \frac{u_{i+1/2}}{12h}\frac{\left(q_1\right)_{i+1}}{\left(q_0\right)_{i+1}} c_{i+2} - \left(-\left(q_1\right)_i \frac{u_{i+1/2}}{12h}\left(2+\frac{\left(q_1\right)_i}{\left(q_0\right)_i}\right) + \right.\\
\left. + \left(q_2\right)_i \frac{u_{i-1/2}}{12h}\frac{\left(q_1\right)_i}{\left(q_0\right)_i} + \left(q_1\right)_i\left(-\frac{u_{i+1/2}}{2h} + k_i^{(1)} + k_i^{(2)}\right)\right) c_{i+1} + \left(-\left(q_1\right)_i \frac{u_{i+1/2}}{12h}\left(2+\frac{\left(q_2\right)_{i+1}}{\left(q_0\right)_{i+1}}\right) + \right.\\
+ \left(q_2\right)_i \frac{u_{i-1/2}}{12h}\left(2+\frac{\left(q_1\right)_{i-1}}{\left(q_0\right)_{i-1}}\right) + \left(q_2\right)_i \frac{u_{i-1/2}}{2h} - \left(q_1\right)_i \frac{u_{i+1/2}}{2h} - \left(\left(q_2\right)_i - \left(q_1\right)_i\right) k_i^{(1)} + \\
\left. + \left(\left(q_2\right)_i + \left(q_1\right)_i\right) k_i^{(2)}\right) c_i - \left(-\left(q_1\right)_i \frac{u_{i+1/2}}{12h}\frac{\left(q_2\right)_i}{\left(q_0\right)_i} + \left(q_2\right)_i \frac{u_{i-1/2}}{12h}\left(2+\frac{\left(q_2\right)_i}{\left(q_0\right)_i}\right) + \right.\\
\left. + \left(q_2\right)_i\left(\frac{u_{i-1/2}}{2h} + k_i^{(2)} - k_i^{(1)}\right)\right) c_{i-1} - \left(-\left(q_2\right)_i \frac{u_{i-1/2}}{12h}\frac{\left(q_2\right)_{i-1}}{\left(q_0\right)_{i-1}}\right) c_{i-2},
\end{gathered} \tag{6}$$

where

$$k_i^{(1)} = \left(\frac{\left(q_1\right)_i}{\left(q_0\right)_i}\left(u_{i+1} - u_{i,}\right) - \frac{\left(q_2\right)_i}{\left(q_0\right)_i}\left(u_i - u_{i-1}\right)\right) / \left(8h\right),$$

$$k_i^{(2)} = \frac{\left(q_1\right)_i}{\left(q_0\right)_i}\frac{u_{i+1}-u_i}{8h} + \frac{\left(q_2\right)_i}{\left(q_0\right)_i}\frac{u_i - u_{i-1}}{8h}.$$

The approximation error of (5) in the case $\left(q_1\right)_{i,j} = \left(q_2\right)_{i,j} = 1$ has the form:

$$\mu_{i+1/2,\,j}\frac{c_{i+1,\,j}-c_{i,\,j}}{h_x^2} - \mu_{i-1/2,j}\frac{c_{i,\,j}-c_{i-1,\,j}}{h_x^2} = \left(\mu_{i,\,j}\left(c_{i,\,j}\right)'\right)' + \mu_{i,\,j}\left(c_{i,\,j}\right)^{(IV)}\frac{h_x^2}{12} +$$

$$+\left(\mu_{i,j}\right)^{''}\left(c_{i,j}\right)^{''}\frac{h_x^2}{4}+\left(\mu_{i,j}\right)^{'}\left(c_{i,j}\right)^{'''}\frac{h_x^2}{6}+\left(\mu_{i,j}\right)^{'''}\left(c_{i,j}\right)^{'}\frac{h_x^2}{6}+O\left(h_x^4\right). \tag{7}$$

Therefore, the operator $\left(\mu c'\right)^{'}-\mu c^{(IV)}h^2/12-\mu''c''h^2/4-\mu'c'''h^2/6-\mu'''c'h^2/6$ must be approximate by a scheme of the second order accuracy for approximation of the diffusion transport operator $\left(\mu c'\right)^{'}$ by a difference scheme of the fourth order of accuracy.
The representative of the diffusion transport operator $\left(\mu c'\right)^{'}$ by a difference scheme of the fourth order of accuracy has the form:

$$\left(q_0\right)_i L\left(c\right)=-A_ic_i+B_{1,i}c_{i+1}+B_{2,i}c_{i-1}+B_{3,i}c_{i+2}+B_{4,i}c_{i-2}. \tag{8}$$

$$B_{1,i}=\left(q_1\right)_i\frac{\mu_{i+1/2}}{h^2}+\left(q_1\right)_i\frac{\mu_{i+1}}{12h^2}\left(\frac{\left(q_1\right)_i}{\left(q_0\right)_i}+2\right)+\left(q_2\right)_i\frac{\mu_{i-1}}{12h^2}\frac{\left(q_1\right)_i}{\left(q_0\right)_i}-\left(q_1\right)_i k_i^{(3)}-\left(q_1\right)_i\frac{\mu''_{i+1}-\mu''_i}{12},$$

$$B_{2,i}=\left(q_2\right)_i\frac{\mu_{i-1/2}}{h^2}+\left(q_1\right)_i\frac{\mu_{i+1}}{12h^2}\frac{\left(q_2\right)_i}{\left(q_0\right)_i}+\left(q_2\right)_i\frac{\mu_{i-1}}{12h^2}\left(\frac{\left(q_2\right)_i}{\left(q_0\right)_i}+2\right)-\left(q_2\right)_i k_i^{(3)}-\left(q_2\right)_i\frac{\mu''_i-\mu''_{i-1}}{12},$$

$$B_{3,i}=-\left(q_1\right)_i\frac{\mu_{i+1}}{12h^2}\frac{\left(q_1\right)_{i+1}}{\left(q_0\right)_{i+1}},\, B_{4,i}=-\left(q_2\right)_i\frac{\mu_{i-1}}{12h^2}\frac{\left(q_2\right)_{i-1}}{\left(q_0\right)_{i-1}}, \tag{9}$$

$$A_i=\left(q_1\right)_i\frac{\mu_{i+1/2}}{h^2}+\left(q_2\right)_i\frac{\mu_{i-1/2}}{h^2}-\left(\left(q_1\right)_i+\left(q_2\right)_i\right)k_i^{(3)}+\left(q_1\right)_i\frac{\mu_{i+1}}{12h^2}\left(\frac{\left(q_2\right)_{i+1}}{\left(q_0\right)_{i+1}}+2\right)+$$

$$+\left(q_2\right)_i\frac{\mu_{i-1}}{12h^2}\left(\frac{\left(q_1\right)_{i-1}}{\left(q_0\right)_{i-1}}+2\right)-\left(q_2\right)_i\frac{\mu''_i-\mu''_{i-1}}{12}-\left(q_1\right)_i\frac{\mu''_{i+1}-\mu''_i}{12}+\left(q_2\right)_i\frac{\mu_{i-1}}{12h^2}\frac{\left(q_1\right)_i}{\left(q_0\right)_i}+$$

$$+\left(q_1\right)_i\frac{\mu_{i+1}}{12h^2}\frac{\left(q_2\right)_i}{\left(q_0\right)_i}-\left(q_1\right)_i\frac{\mu_{i+1}}{12h^2}\frac{\left(q_1\right)_{i+1}}{\left(q_0\right)_{i+1}}-\left(q_2\right)_i\frac{\mu_{i-1}}{12h^2}\frac{\left(q_2\right)_{i-1}}{\left(q_0\right)_{i-1}},$$

where

$$k_i^{(3)}=\frac{\left(q_1\right)_i}{\left(q_0\right)_i}\frac{\mu_{i+1}-\mu_i}{4h^2}-\frac{\left(q_2\right)_i}{\left(q_0\right)_i}\frac{\mu_i-\mu_{i-1}}{4h^2},\,\mu''_i=\left(\frac{\left(q_1\right)_i}{\left(q_0\right)_i}c_{i+1}-2c_i+\frac{\left(q_2\right)_i}{\left(q_0\right)_i}c_{i-1}\right)/h^2.$$

4. Solution method of grid equations

The discrete analog of the model (1) – (3) in the operator form is the following [19, 23, 25]:

$$Au=f \tag{10}$$

with the nondegenerate operator A, defined in the real Hilbert space H. Let us consider the implicit two-layer iterative scheme in the form:

$$B\frac{y_{k+1}-y_k}{\tau_k}+Ay_k=f,\, k=0,1,\ldots \tag{11}$$

with an random initial approximation $y_0\in H$ and the nondegenerate operator B [23]. Any two-layer iterative method based on the scheme (10) is characterized by the operators A and B, the energy space H_D, in which we prove the convergence of the method, and set the iteration parameters τ_k. The main question in the theory of iterative methods is the issue of the optimal choice of the parameter τ_k [19].

No a priori information about the operators of the scheme (11) (in addition to the conditions of the general form$A=A^*>0$, $\left(DB^{-1}A\right)^*=DB^{-1}A$, etc.) not requires in two-layer iterative methods of variational type for the calculation parameters τ_k. The construction of these methods is based on the following principle: if the approximation y_k is set, and y_{k+1} is defined according to the scheme (8), the iterative parameter τ_{k+1} is chosen from the condition of minimum in H_D of the error $z_{k+1} = y_{k+1} - u$, where u is the solution of problem (10). The sequence y_k, defined according to the formula (11) (the parameters are selected from the above conditions), is minimal for a quadratic functional of the form $I\left(y\right) = \left(D\left(y-u\right), y-u\right)$. This functionality (because of positive deniteness of the operator D) is below limited, and reached a minimum, equal to the zero, at the solution of equation (10), i.e. at $y = u$. The parameter selection τ_{k+1} of the specified condition provides a local minimization of the functional $I\left(y\right)$ at the converting from y_k to the y_{k+1}, i.e. for one iteration step. In the case of explicit schema ($B = E$) the conversion from y_k to the y_{k+1} is carried out according to the formula:

$$y_{k+1} = y_k - \tau_{k+1} r_k, r_k = Ay_k - f.$$

For self-adjoint positive definite operator the conversion from y_k to the y_{k+1} is carried out in the direction of $-r_k$, which coincides with the direction of antigradient for functionality $\left(A\left(y-u\right), y-u\right)$ at the point y_k. The greatest decreasing of the functionality value is carried out in the direction of antigradient. The parameter τ_{k+1} is defined from the condition of minimum in H_D of the error $z_{k+1} = y_{k+1} - u$ [25].

The formula for calculation of the iterative parameter τ_{k+1} was defined according to the assumption that the operator A is the nondegenerate.

Firstly, the error equation for $z_k = y_k - u$, $k = 0, 1, \ldots$ is defined. Then we substituted $y_k = \tau_k + u$ to the scheme (8), and obtained the next expression: $z_{k+1} = \left(E - \tau_k B^{-1} A\right) z_k$, $k = 0, 1, \ldots$, $z_0 = y_0 - u$. Due to the substitution $z_k = D^{-\frac{1}{2}} x_k$, we obtained the equation, which contained only one operator:

$$x_{k+1} = S_{k+1} x_k, S_k = E - \tau_k C, C = D^{-\frac{1}{2}} \left(DB^{-1}A\right) D^{-\frac{1}{2}}. \tag{12}$$

The defined above problem about the selection of the parameter τ_{k+1} was formulated using the equation $\|z_k\|_D = \|x_k\|$ ($\|\cdot\|_D$ is the norm in H_D, $\|\cdot\|$ is the norm in H): the parameter τ_{k+1} is chosen from the condition of the norm minimum x_{k+1} in the space H. The norm is calculated as following:

$$\begin{gathered} \|x_{k+1}\|^2 = \left(\left(E - \tau_{k+1} C\right) x_k,\, \left(E - \tau_{k+1} C\right) x_k\right) = \|x_k\|^2 - 2\tau_{k+1} \left(Cx_k, x_k\right) + \tau_{k+1}^2 \left(Cx_k, Cx_k\right) = \\ = \left(Cx_k, Cx_k\right) \left[\tau_{k+1} - \left(Cx_k, x_k\right) / \left(Cx_k, Cx_k\right)\right]^2 + \|x_k\|^2 - \left(Cx_k, x_k\right)^2 / \left(Cx_k, Cx_k\right). \end{gathered} \tag{13}$$

The operator C is the nondegenerate as the operator A is the nondegenerate. So, we obtained the condition $\left(Cx_k, Cx_k\right) > 0$ for each x_k, and the minimum of norm is achieved at the next condition:

$$\tau_{k+1} = \left(Cx_k, x_k\right) / \left(Cx_k, Cx_k\right). \tag{14}$$

The equation (13) substituted to the (14):

$$\|x_{k+1}\| = \rho_{k+1} \|x_k\|, \tag{15}$$

where

$$\rho_{k+1}^2 = 1 - \left(Cx_k, x_k\right)^2 / \left\{\left(Cx_k, Cx_k\right)\left(x_k, x_k\right)\right\}. \tag{16}$$

The formula (14) is defined the optimal value of the iterative parameter τ_{k+1}. The expression $x_k = D^{-\frac{1}{2}} z_k$ is substituted to the (14):

$$\tau_{k+1} = \left(DB^{-1}Az_k, z_k x_k\right) / \left(DB^{-1}Az_k, B^{-1}Az_k\right), \; k = 0, 1, \ldots. \tag{17}$$

According to the $Az_k = Ay_k - Au = Ay_k - f = r_k$ is the residual vector, and $B^{-1}r_k = \omega_k$ is correction vector, the formula for parameter τ_{k+1} has the form:

$$\tau_{k+1} = (D\omega_k, z_k) / (D\omega_k, \omega_k), \; k = 0, 1, \ldots, \tag{18}$$

and the iterative scheme (11) in the explicit formula for calculation y_{k+1} is the following:

$$y_{k+1} = y_k - \tau_{k+1}\omega_k, k = 0, 1, \ldots. \tag{19}$$

The algorithm, implemented the proposed method, is the following:

1) residual vector $r_k = Ay_k - f$ is calculated according to the given y_k;
2) equation for the correction vector $B\omega_k = r_k$ is calculated;
3) parameter τ_{k+1} is obtained according to the formula (18);
4) new approximation y_{k+1} is calculated according to the formula (19).

We considered the special cases of two-layer gradient methods, which we will use to solve the model problem (1) – (3). Each specific method is determined by the choice of the operator D and has its area of applicability. The operator D was chosen so that only known in the iteration process values were in formula (18) for the iterative parameter τ_{k+1}.

If the operator A is self-adjoint and positively defined in H, then the method of steepest descent (MSS) can be used for solution (11). If the operator A is non-selfadjoint and nondegenerate, and the operator B^*A positively identified, then we can use the minimal residual method (MRM) [34].

Let us consider that the operator A is self-adjoint and positively defined in H. For the method of steepest descent (MSS) $D = A$. The operator B must be positively defined in H.

According to the ratio $Az_k = Ax_k - f = r_k$ and $A = A^*$ from the (15) we get the formula for iterative parameter τ_{k+1} in the implicit method of steepest descent:

$$\tau_{k+1} = (r_k, \omega_k) / (A\omega_k, A\omega_k), k = 0, 1, \ldots. \tag{20}$$

For the case of explicit two-layer scheme (14) $B = E$ we get $\omega_k = B^{-1}r_k = r_k$, and the formula for the τ_{k+1} has the form:

$$\tau_{k+1} = (r_k, r_k) / (Ar_k, Ar_k), k = 0, 1, \ldots. \tag{21}$$

The parameter τ_{k+1} is calculated according to the formula (21) at the solution of model problem (1) – (3) using the method of steepest descent in the residual, and the minimal residual method in the correction is implemented according to the calculation formula (20).
The method of minimum corrections (MMP) can be used to solve the model problem (1) – (3) in the case of any non-degenerate non-selfadjoint operator A, it also requires the positive definiteness of the operator B^*A. The condition for the method of minimal residuals is the following: $D = A^*A$.
The formula for the iterative parameter τ_{k+1} in the method of minimum corrections is the following:

$$\tau_{k+1} = (A\omega_k, r_k)/(A\omega_k, A\omega_k), k = 0, 1, \ldots. \tag{22}$$

For the case of the explicit scheme (11) ($B = E$) the operator A, must be positively defined, and the formula for the τ_{k+1} has the form:

$$\tau_{k+1} = (Ar_k, r_k) \,/\, (Ar_k, Ar_k)\,, k = 0, 1, \ldots. \tag{23}$$

The analysis of the effectiveness of the above-described gradient methods of variational type was performed on the basis of the numerical solution of the model problem (1) – (3). Comparison results of convergence speeds of two-layer gradient methods of variational type, which is used to solve the problem of the form (1) – (3): the minimum residuals method (MRM), the minimum correction method (MMC), the steepest descent method in the residual (MSDR), the steepest descent method in the corrections (SDMC) were given in Table [35].

Table

Comparison of convergence velocities of the methods of variational type

MRM				
$n \setminus p$	1	2	3	4
1	$I_1 = 44, I_2 = 48$	$I_1 = 43, I_2 = 47$	$I_1 = 43, I_2 = 47$	$I_1 = 42, I_2 = 48$
2	$I_1 = 48, I_2 = 53$	$I_1 = 47, I_2 = 52$	$I_1 = 46, I_2 = 53$	$I_1 = 45, I_2 = 53$
3	$I_1 = 21, I_2 = 21$	$I_1 = 20, I_2 = 22$	$I_1 = 19, I_2 = 22$	$I_1 = 19, I_2 = 21$
4	$I_1 = 15, I_2 = 17$	$I_1 = 13, I_2 = 14$		
MMC				
1	$I_1 = 65, I_2 = 83$	$I_1 = 64, I_2 = 77$	$I_1 = 63, I_2 = 76$	$I_1 = 62, I_2 = 76$
2	$I_1 = 73, I_2 = 87$	$I_1 = 73, I_3 = 87$	$I_1 = 72, I_2 = 86$	$I_1 = 71, I_2 = 85$
3	$I_1 = 51, I_2 = 51$	$I_1 = 46, I_2 = 50$	$I_1 = 43, I_2 = 49$	$I_1 = 40, I_2 = 48$
4	$I_1 = 38, I_2 = 40$	$I_1 = 35, I_2 = 38$		
MSDR				
1	$I_1 = 62, I_2 = 69$	$I_1 = 61, I_2 = 67$	$I_1 = 60, I_2 = 67$	$I_1 = 59, I_2 = 67$
2	$I_1 = 68, I_2 = 71$	$I_1 = 67, I_2 = 74$	$I_1 = 67, I_2 = 74$	$I_1 = 66, I_2 = 74$
3	$I_1 = 32, I_2 = 35$	$I_1 = 30, I_2 = 35$	$I_1 = 29, I_2 = 35$	$I_1 = 28, I_2 = 34$
4	$I_1 = 23, I_2 = 24$	$I_1 = 20, I_2 = 21$		
SDMC				
1	$I_1 = 64, I_2 = 68$	$I_1 = 67, I_2 = 79$	$I_1 = 66, I_2 = 78$	$I_1 = 65, I_2 = 78$
2	$I_1 = 74, I_2 = 86$	$I_1 = 73, I_2 = 87$	$I_1 = 73, I_2 = 86$	$I_1 = 72, I_2 = 86$
3	$I_1 = 49, I_2 = 53$	$I_1 = 45, I_2 = 52$	$I_1 = 42, I_2 = 51$	$I_1 = 41, I_2 = 49$
4	$I_1 = 38, I_2 = 40$	$I_1 = 35, I_2 = 36$		

In Table: p is an iteration number; n is the number of time layer; I_k is the amount of iterations, which are necessary for the convergence; the method for solution SLAE for equation, described the change of concentration P_k, $k \in \{1, 2\}$.

The minimum correction method (MMC) was chosen as the main method due to its fastest convergence speed, as shown in table.

5. Parallel variant of method for solution grid equations

The *k-means* method was used for geometric partition of the calculation domain for uniform loading of MCS processors. This method is based on the minimization of the functional of the total sample variance of the elements scatter (nodes of the computational grid) relative to the gravity center of subdomains: $Q = Q^{(3)}$.

Let X_i be the set of computational grid nodes, included in the i-th subdomain, $i \in \{1, ..., m\}$, m is the given number of subdomains.

$$Q^{(3)} = \sum_i \frac{1}{|X_i|} \sum_{x \in X_i} d^2(x, c_i) \to \min,$$

where $c_i = \frac{1}{|X_i|} \sum_{x \in X_i} x$ is the center of the subdomain X_i, and $d(x, c_i)$ is the distance between the calculated node and the center of the grid subdomain in the Euclidean metric. The *k-means* method converges only when all subdomain will be approximately equal.

Result of using the k-means method for model of two-dimensional and three-dimensional areas of regular shape is given in Fig. 2.

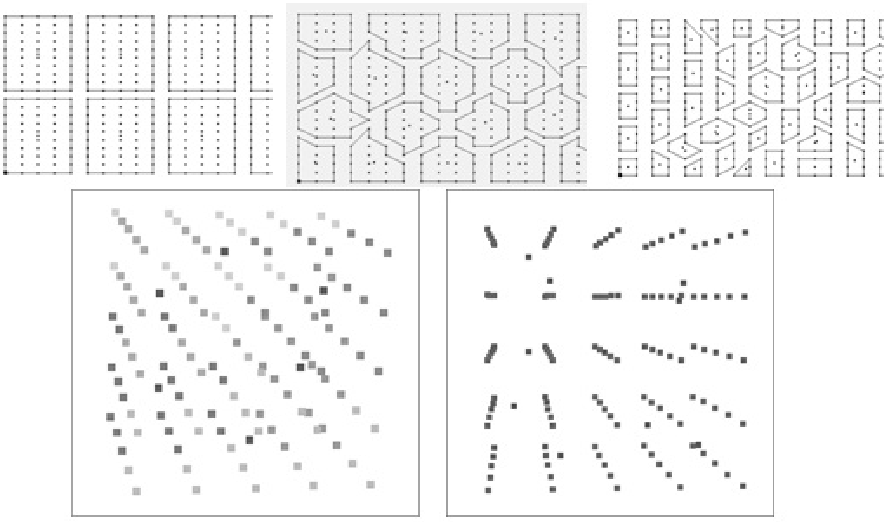


Fig. 2. Results of using k-means method for splitting the model domains with the correct form

For data exchange in the computation process it is required to find all points on the border of each subdomain. For this purpose, the Jarvis algorithm (the problem of constructing the convex hull) was used. A list of neighboring subdomains for each subdomain was formed, and the algorithm of data transfer between the subdomains was developed.

The doubling method was used in solution of the SLAE of the minimal correction method for calculation of the iterative parameter τ.

The algorithm synchronization for solving the problem (1) – (3) is required only in the MMC during the transition to the next iteration.

6. Software complex description

The software complex was performed for solution of the model problem of plankton interaction, and populations of the ctenophores *Mnemiopsis leidyi* and *Beroe ovata* predators in the form (1) – (3), which calculated the concentrations of the major aquatic organisms, including phyto - and zooplankton, jelly microplankton (species – ctenophores) in the areas of complex shape (the Azov Sea and Taganrog Bay) [14, 34, 35].

The developed software complex can be as follows:

1) development and implementation of integrated fisheries monitoring in the waters (monitoring, evaluation and prediction of ecosystems situation, fodder stocks and fishing objects);

2) development, negotiation of proposals and measures for ensuring an optimal regime, biodiversity, fisheries resources, ecosystems of shallow waters;
3) improvement of environmental research methodology, development of new methodology, testing and implementation of promising research methods of water ecosystems and individual components;
4) development and improvement of methods of diagnostics of toxic effect of biogenic substances on hydrobionts, including early and differential diagnostics of toxicity, as well as fundraising antidote for the protection of aquatic ecosystems;
5) organizing and conducting research to identify trends and patterns of change in the state of aquatic ecosystems under anthropogenic impact, development of proposals and measures to reduce and prevent such effects;
6) assessment of damages to fisheries caused by different economic activities, development of proposals for the prevention, reduction and adequate compensation of damages.

The velocity field of water flow, calculated in [21], is the input data for used models of biological kinetics. Consistently gathering rectangular grid with dimensions: $251 \times 351 \times 15$, $502 \times 702 \times 30$, $1004 \times 1404 \times 60$, $\ldots$ was used for mathematical modeling hydrological and hydrodynamic processes in the three dimensional space of complex form – the Azov Sea and the Taganrog Bay.

The initial distribution of polluting nutrients, phyto-, zooplankton and ctenophores were taken into account in the form corresponding to the space-time scales of simulated processes. The implemented algorithm of the numerical solution allows us freely change the boundaries, the view of control functions and values of the corresponding parameters [27]. The functioning of the system and the knowledge of its basic characteristics allow using the phenomenological approach in the program configuration. The efficiency of such approach is quite high because the system behavior is often determined by the precision of individual parameters and their correlations.

The calibration and verification of the proposed model (1) – (3) for shallow water have been performed on the basis of ecological data of the Azov Sea, obtained at the scientific-research expeditions by the scientists of the Southern Federal University, the Southern Scientific Center of the Russian Academy of Science (SSC RAS), the Azov Scientific-Research Institute of Fisheries (AzSRIF) since 2000. Processing of the expidition data consists of the digitizing, conversion into standard units, classification for use in various model problems of sea hydrobiology.

The developed software complex includes the control unit, oceanographic and meteorological databases, interface systems, input-output and visualization systems. The software complex has the user-friendly interface that allows us to enter the necessary information in the dialog. The developed software complex is universal, and as a rule, linking it to specific objects and areas is carried out at the level of the input information. It means that the special information database, containing information about physical-geographical and climatic conditions of researched objects, the parameters for determining the sources of impurities, is required for practical using of units.

The high-level language C++ was used at the development of the software complex. The messaging technology (MPI) was used for clusters.

7. Results of numerical experiments

External frequency, leading to complication of the system, was taken into account at modeling the spatially-heterogeneous processes of interaction of the main aquatic organisms of the Azov Sea (1) – (3). Thus fluctuations in the density of plankton can be so great that cannot be explained by random fluctuations, and the visual picture is so that a relatively small area of high density (“spots”, “clouds”) are separated by spaces with low densities, sometimes not fixed by standard methods of observation. This phenomenon is especially clearly expressed in water areas, which are characterized by the necessity for nutrient elements. The vegetative period of phytoplankton were taken into account in the modeling of biological kinetics processes.

Various systems of the “Analytical GIS” portal, developed by the Institute of Information Transmission Problems of RAS (IITP RAS, Moscow) [6] and the data of SRC “Planeta” (see Fig. 3, Fig. 4), site data of the AzSRIF were used as the input data for modeling of hydrophysical processes in addition to expidition data, literary sources.

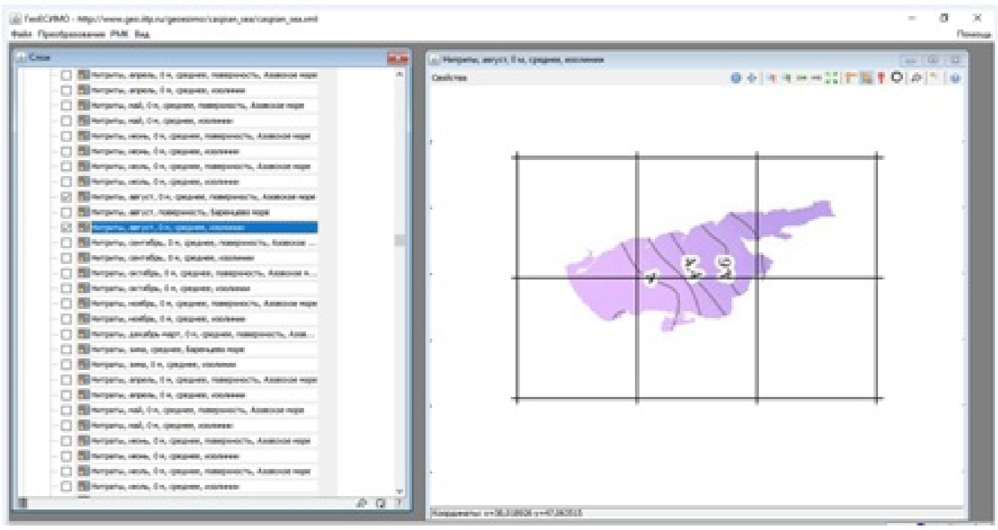
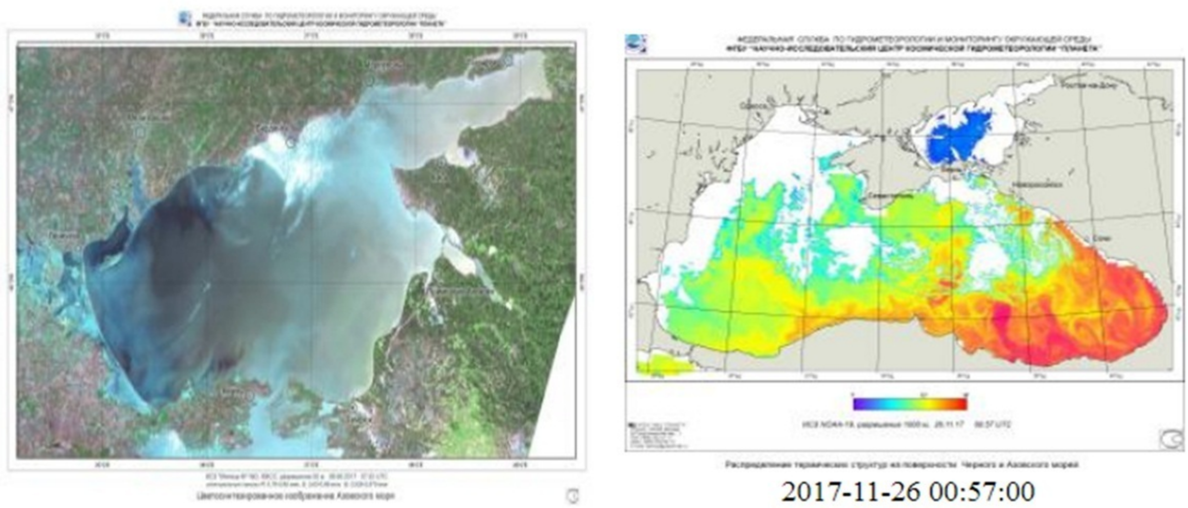


Fig. 3. Navigation board of the “Analytical GIS” portal



a) Satellite image of the Azov Sea b) Map of sea surface temperatures

Fig. 4. Ecological data from the site of the SRC “Planeta”

Diffusion processes in the water occurred in the direction of smoothing the spatial distribution and dispersion of the “patches” of phytoplankton. One of the attempts to explain the paradox of stability “spots” with the help of numerical experiments is to assume about the active movement of heterotrophic organisms (zooplankton and fish) in the direction of the “food” gradient that provides the consolidation of spatial heterogeneity of nutrients in the aquatic environment. Sustainable heterogeneity of the spatial distribution can be, for example, due to diffusion processes and the presence of phytoplankton mechanism of actoring regulation, i.e. regulation of the growth rate through selection in the environment of biologically active metabolites.

Calculation results of pollution of biogenic matter for the model (1) – (3) (the initial distribution of the current fields of water flow in the Northern wind) are given in Fig. 5. The influence of current structures of the water flow in the Azov Sea on the distribution of pollution nutrients and phytoplankton is given in Fig. 5, 6 below.

The modeling results of phytoplankton dynamics in the Azov Sea are given in Fig. 6 (N is the iteration number, the initial distribution of the current fields of water flow in the Northern wind).

The maximum values of concentrations of biogenic matter (nitrogen) and the phytoplanktons are defined by the white color, and the minimum values of concentrations – by the black color.

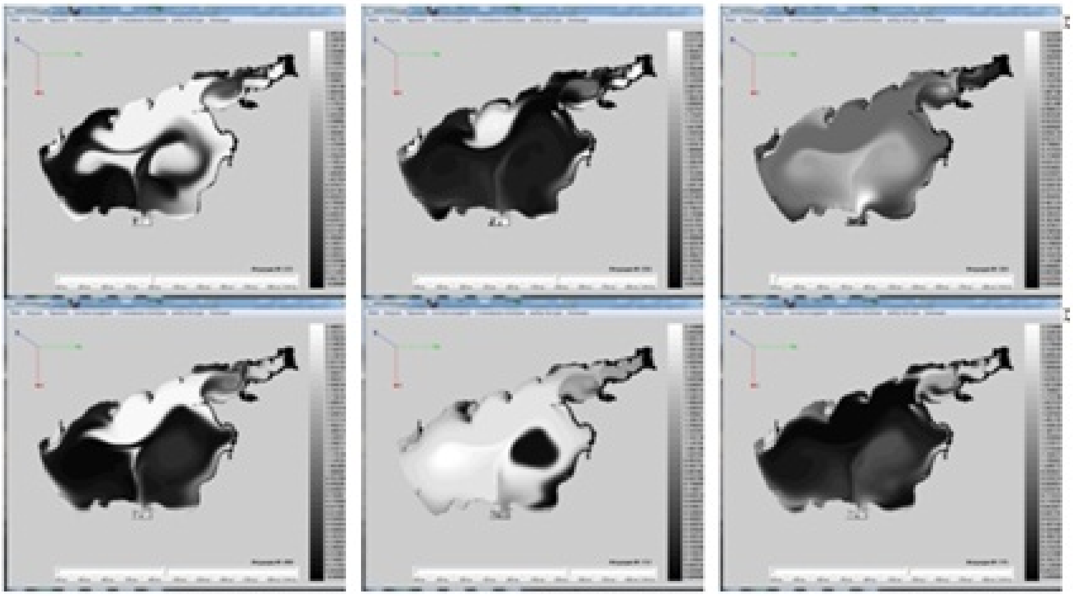


Fig. 5. Distribution of pollutant concentrations in different time

The criterion for the adequacy of the proposed model of the plankton and ctenophores interaction in (1) – (3) of shallow water was the error estimation of modeling with the field data according to the available measurements, which was calculated by the formula: $\delta = \sqrt{\sum_{k=1}^{n} (S_{k\ nat} - S_k)^2} \Big/ \sqrt{\sum_{k=1}^{n} S_{k\ nat}^2}$, where $S_{k\ nat}$ is the concentration value obtained with the help of field expedition measurements; S_k is the value calculated with the help of the proposed model. The concentrations of pollutions and plankton, calculated at the various wind situations, were taken into account if the relative error did not exceed 30%.

The analysis of various development ways of the Azov Sea ecosystem was performed using the mathematical modeling. According to the obtained results, the *B. Ovata* can control the coastal populations of the *Mnemiopsis leidyi.*

Using the model data, we can research and predict the growth of the *M.Leidyi* population in spring and summer and the disappearance of species after the appearance of the *B.ovata.* In addition, it is possible to observe dynamics of changes in the biomass of fodder zooplankton in coastal waters and the open sea, the intense reproduction of bivalves, which declined in recent

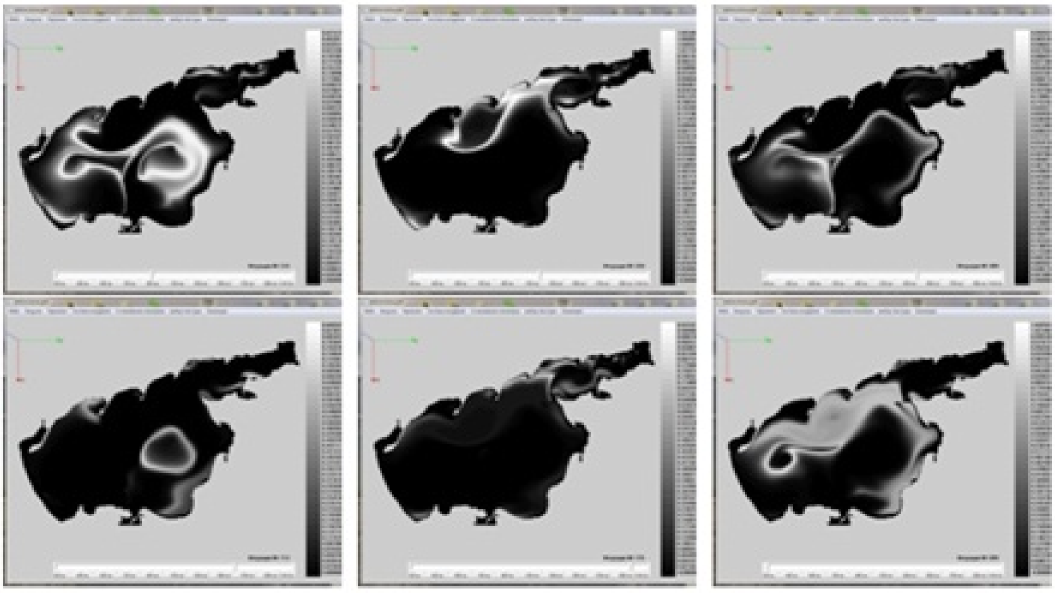


Fig. 6. Distribution of phytoplankton concentrations in different time

years under the influence of introduction of *M. leidyi*, and the process of restoration of benthic communities.

Further development of the Azov-Black Sea basin ecosystem depends on the stability of the introduction of *B. ovata*. In this sense, it is necessary to mention that the difference in climatic conditions and water circulation has had a negative impact on the development of *B. ovata* in recent decades. Early appearance of *B. ovata* in a relatively cold period reflects its lasting adaptation to the conditions of the Azov-Black Sea basin. Possible development of this species in early summer may stop the outbreak of *M. leidyi*, which has become a regular in July-August.

According to the obtained results of numerical experiment, the appearance of the *M. Leidyi* ctenophore led to irreversible major changes at all levels – from individual populations to the ecosystem of the sea. Stable moving of *B. ovata* leads to the restoration of fodder zooplankton, benthic communities and fish populations that feed on plankton in the Azov-Black Sea basin.

Significant structural and functional reorganization of ecosystems occurred as the result of the activity of *M. leidyi* that has the influence on the production and destruction processes.

Favorable conditions for the development of heterotrophic microplankton are formed at the region of active development of *M. leidyi* due to continuous discharge of large amounts of mucus. The heterotrophic microplankton is processing this organic. And as a result the structure and concentration of biogenic elements are changed, that is an additional stimulant for the development of phytoplankton community.

Conclusion

The primary verification of the model of the Azov Sea ecosystem was performed with the help of expedition researches. Modeling and prediction problem of the water ecosystem situation in the Azov Sea under anthropogenic influence and invasions of ctenophores for a comprehensive research of this unique water object was implemented. The software complex, combining the mathematical models and databases, was performed with the help of which the conditions of invasion of invasive species (ctenophores) on the changing of the concentration of phyto- and zooplankton.

The distinctive features of the developed algorithms are the decomposition of calculations based on the method of k-means method, implementing a set of hydrobiological model problem on a multiprocessor computing system, and the high performance, reliability and accuracy of results. High performance is achieved through the use of efficient numerical methods for solving grid equations, oriented for using on parallel computer systems in real and accelerated time. The accuracy is achieved by taking into account the important physical factors, such as the Coriolis force, turbulent exchange, the complex geometry of bottom and coastline, evaporation, river flows, the dynamic reconstruction of computational domain, wind stress and friction on the bottom and the deviation of pressure field from the hydrostatic approximation. The accuracy is achieved by using detailed computational grids, taking into account the degree of "fillness" of computational cells, and the absence of nonconservative dissipative terms and revision sources (sinks), occurring from finite difference approximations.

The comparison of developed software complex for MCS, implementing the development scenarios of the environmental situation in the Azov Sea with using a numerical implementation of the developed model of biological kinetics, with similar works in the sphere of mathematical modeling of hydrological processes was performed. As a result, the prediction accuracy of changes in concentrations of pollutants, phyto- and zooplankton, and ctenophores was increased by 10 – 20 % depending on the model problem of aquatic ecology.

In the future we plan to consider the impact of pollution nutrients subsidence, causing phytoplankton blooms in summer and coming out of the air with subsequent deposition on the water surface, in the model of plankton and ctenophores interaction.

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ПРАКТИЧЕСКИЕ АСПЕКТЫ РЕАЛИЗАЦИИ ПАРАЛЛЕЛЬНОГО АЛГОРИТМА РЕШЕНИЯ ЗАДАЧИ ВЗАИМОДЕЙСТВИЯ ПОПУЛЯЦИЙ ГРЕБНЕВИКА В АЗОВСКОМ МОРЕ

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Целью работы является разработка и исследование математической модели процессов взаимодействия популяций планктона и гребневика на основе современных информационных технологий и вычислительных методов, позволяющей повысить точность прогнозного моделирования экологической обстановки мелководного водоема в летний период. Модель учитывает: движение водного потока; микротурбулентную диффузию; нелинейное взаимодействие популяций планктона и гребневика; биогенный, температурный и кислородный режимы; влияние солености. Использование метода частичной заполненности расчетных ячеек при дискретизации модели позволяет существенно повысить точность и сократить время вычислений. Практическая значимость работы состоит в том, что предложенная модель программно реализована, определены пределы и перспективы ее практического использования. На базе супер-ЭВМ разработано экспериментальное программное обеспечение, предназначенное для математического моделирования возможных сценариев развития экосистем мелководных водоемов на примере Азовского моря в летний период. При параллельной реализации были использованы методы декомпозиции сеточных областей для вычислительно-трудоемких задач диффузии-конвекции, учитывающие архитектуру и параметры многопроцессорной вычислительной системы.

Ключевые слова: математическая модель, гидробиологические процессы, экспедиционные исследования, гребневик, Азовское море, параллельный алгоритм, многопроцессорная вычислительная система.

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