Краткие сообщения
Brief reports

Brief report
DOI: 10.14529/ctcr240310

MODELLING POLITICAL PROCESSES IN A MULTICRITERIAL SETTING

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Abstract. A political process is a complex management system that includes, for instance, economics, electoral processes, and ecology. To make decisions when managing such a system, it is necessary to use some rules that allow to intentionally choose the best or an acceptable option. Such rules are called performance criteria. This paper discusses problems that affect different aspects of the management systems of political processes, and when choosing a solution, it is necessary to evaluate options using several criteria. These are called multicriteria problems. They emerge in strategic planning of a system, its forecasting and development. **The goal of the study** is to consider methods and tools of solving multicriteria problems arising while managing political processes. We discuss mathematical methods that help discard solutions, which are, in all respects, worse than others, and choose a compromise option from the remaining ones. **Methods.** We analyzed various methods of choosing a compromise solution, their advantages and disadvantages, and identified their areas of use. First, we consider various options for condensing the criteria and illustrate their positive and negative sides. Then, we describe the procedure for constructing a set of Pareto optimal solutions. Another method is the method of successive concessions, which is a procedure for choosing a solution in a dialogue (interactive) setting. Finally, we describe a decision-making procedure based on the analytic hierarchy process. **Results.** The paper shows the effectiveness of applying mathematical methods to solve multicriteria problems in managing political processes. The paper concludes with an example of a solution of a management problem in accordance with all the requirements we consider. **Conclusion.** The use of mathematical modeling and methods to solve multi-criteria management problems helps politicians make efficient decisions in their work and provides them with communication tools by using professional mathematical language.

Keywords: mathematical modeling, multiplicity of efficiency criteria, Pareto efficiency, analytic hierarchy process

For citation: Gelrud Ya.D., Shestakova L.I., Gusev E.V., Kodkin V.L., Shiriaev V.I. Modelling political processes in a multicriterial setting. *Bulletin of the South Ural State University. Ser. Computer Technologies, Automatic Control, Radio Electronics*. 2024;24(3):111–123. DOI: 10.14529/ctcr240310

Краткое сообщение
УДК 32:001.8; 32:303.8
DOI: 10.14529/ctcr240310

МОДЕЛИРОВАНИЕ ПОЛИТИЧЕСКИХ ПРОЦЕССОВ В УСЛОВИЯХ МНОГОКРИТЕРИАЛЬНОСТИ

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Аннотация. Политические процессы представляют собой сложную систему организационного типа. Чтобы принять решение при управлении такой системой, необходимо использовать некоторые правила, позволяющие целенаправленно выбрать наилучший или допустимый вариант действий. Такими правилами являются критерии эффективности. В данной статье рассматриваются задачи, затрагивающие различные аспекты деятельности систем управления политическими процессами, причем при выборе решения возникает необходимость оценивать варианты, используя несколько критериев. Задачи такого рода называются многокритериальными. Они возникают при стратегическом планировании, прогнозировании и развитии системы. **Цель исследования.** Основной целью статьи является рассмотрение методов и средств решения многокритериальных задач при управлении политическими процессами. Рассмотрен математический аппарат, который помогает отбросить заведомо худшие варианты решений, по всем показателям уступающие другим, и выбрать из оставшихся компромиссный вариант. **Материалы и методы**. Проведен анализ различных методов выбора компромиссного решения, выявлены их достоинства и недостатки, определены области использования. Рассмотрены различные варианты свертки критериев, проиллюстрированы их положительные и отрицательные стороны. Описана процедура построения множества эффективных решений, оптимальных по Парето. Рассмотрена процедура выбора решения в диалоговом (интерактивном) режиме, использующая метод последовательных уступок. В заключение описана процедура принятия решений на основе метода анализа иерархий. **Результаты.** В статье показана эффективность применения математических методов решения многокритериальных задач в управлении политическими процессами. В конце статьи рассматривается пример решения управленческой задачи в соответствии со всеми перечисленными требованиями. **Заключение.** Использование математического моделирования и методов для решения многокритериальных управленческих задач в профессиональной деятельности политика позволяет повысить эффективность принимаемых им решений и обеспечивает его коммуникационными средствами за счёт использования профессионального математического языка.

Ключевые слова: математическое моделирование, множественность критериев эффективности, оптимальность по Парето, метод анализа иерархий

Для цитирования: Modelling political processes in a multicriterial setting / Ya.D. Gelrud, L.I. Shestakova, E.V.Gusev et al. // Вестник ЮУрГУ. Серия «Компьютерные технологии, управление, радиоэлектроника». 2024. Т. 24, № 3. С. 111–123. DOI: 10.14529/ctcr240310

Introduction

Despite the complex nature of the tasks of managing political processes, politicians have been going on by trial and error for a long time, relying on their mind, common sense and intuition. Mind and intuition are by all means important, but at present the complexity of political processes, their intensity, the social price of wrong decisions stimulate the authorities to be guided by a more modern and reliable methodology when making decisions. The achievements of foreign authors in this field are described in sufficient detail in [1, 2]. Unfortunately, we have to state that the achievements of Russian scientists in the area in question are significantly more modest. Firstly, our country began to implement mathematical methods and informatization into managing complex organizational systems later than others.

In addition, dogmatism, inherited from the Soviet time, and, all too often, absurd secrecy in political decision-making created enormous difficulties for studying and implementing methods of modeling into political work.

Nevertheless, the findings of recent years [3–6] should be regarded as a significant contribution into the development of the methodology of political and social sciences.

In political science as an area of organizational management, management has specific characteristics associated with high risks, uncertainty and incomplete information. In this area, it is impossible to conduct experiments to obtain an optimal solution, since political and social environment cannot be returned to its original state due to the irreversibility of political processes. Therefore, it is necessary to use mathematical modeling, which consists in studying not the object itself, but a mathematical problem that is in objective correspondence with the object being analyzed, in order to obtain the necessary information about it. Mathematical models and their application for solving management problems in social, economic and organizational systems are presented in [7–10]. This article discusses issues that affect different aspects of political process management systems, and when choosing a solution, it is necessary to evaluate options using several criteria. Such tasks are called **multicriterial**. They emerge in strategic planning, forecasting and development of a system. The examples of multicriterial problems described in literature are mainly related to economic systems [11–15], but we will analyze methods of multicriterial optimization and make recommendations for their use in political science. Let us consider, for example, the development program of a certain region. What is specifically meant by effective development in this case? What criterion should be used to choose the solution? First of all, it is desirable to maximize the number of jobs, average wages, and the gross output of enterprises in the region. It would also be beneficial to improve the environmental situation and the quality of consumer services, increase the level of self-government and political activity, etc.

As for budget expenditures, it would be a good idea to minimize them, and maximize the well-being of the region's population. Additional criteria may arise when solving this problem.

Such multiplicity of efficiency criteria (let us denote them as F_1, F_2, …, F_n), taking numerical values that are desirable to maximize or minimize, is called *multicriteriality*. This situation is quite typical when modeling political processes.

Is it possible to find a solution that meets all the criteria at once? The problem is not solved generally. The criteria are mostly contradictory, and finding the extremum of one does not entail simultaneous conversion to the extremum of the others. Therefore, the often declared slogan ‘to achieve the maximum effect at a minimum cost’ is a false phrase which cannot be of scientific interest.

How, if necessary, to evaluate efficiency and develop a solution with several criteria? Let us discuss the appropriate methods in this article.

The article has the following structure. In addition to the Introduction section, the first section presents the methods of combining many criteria into one integrated one which are called methods of aggregating criteria, section 2 discusses the concept and procedure for choosing Pareto-optimal solutions. Section 3 provides a dialog method for solving multi-criteria problems. The fourth section sets out the method of successive assignments. Section 5 discusses decision-making processes based on the hierarchy analysis method, and details an example of how to solve the problem of selecting a project when developing and implementing a program of execution of a national project in a federal subject. In conclusion, recommendations are made on the use of multicriterial optimization methods to solve problems of political process management.

1. Criteria aggregation methods

There are many ways to lead a multi-criteria problem to a single-criterion problem, at the same time a generalized (integral) function of given criteria, which is considered as a decision criterion, is defined. Often such a criterion is written as a fraction, in the numerator of which there are values that need to be increased, and in the denominator – those that need to be reduced. For example, the average salary, he quality of consumer services of the population – in the numerator, and budget expenditures – in the denominator.

The method of such aggregation of criteria does not provide an optimal solution, as when applied, it is assumed that the deterioration under one criterion is compensated by improving the other; in the general case, this is not true. As an example, let us consider the criterion that Leo Tolstoy proposed for assessing a person. This criterion is presented in the form of a fraction, in the numerator of which there are objective assessments of human merits, and in the denominator – a subjective assessment (estimates are set, for example, according to the 5-point system).

$$\text{Criterion for assessing a man's merits} = \frac{\text{objective merits of a man}}{\text{their opinion of themselves}}. \quad (1)$$

At first glance, this approach seems logical. But if a person's assessment of their merits is not fair, while their opinion of themselves is even lower, then Leo Tolstoy's criterion will have great value!

We see that the use of the criterion in the form of a fraction (1) can lead to paradoxical conclusions.

Many use a different way of aggregating performance criteria, forming a ‘weighted sum’ in which each F_i criterion enters with a ‘weight’ q_i corresponding to its degree of importance:

$$\mathrm{F} = q_1F_1 + q_2F_2 + \ldots + q_nF_n, \quad (2)$$

where weight $q_i > 0$ at F_i maximization, and weight $q_i < 0$ at F_i minimization.

With the arbitrary assignment of weights q_1, q_2, …, q_n, this method also cannot provide an objectively optimal solution. A person, making a decision in accordance with this criterion, must first attribute ‘weight coefficients’ to different indicators, but they depend on a person's preferences and can change according to the situation.

Let us illustrate this with an example. Rushing to work in the morning, a person considers the following options: taking a bus is cheap but takes a long time; taking a taxi is faster but the cost is significantly higher.

It makes a typical two-criteria decision-making problem. At the same time, it is necessary to minimize the first criterion, the cost of commuting P, and it is also necessary to minimize the second criterion, commuting time T. But these criteria are incompatible, when one decreases, the other one increases, but a compromise solution acceptable on both criteria must be reached. When making a decision, a person subconsciously uses a generalized indicator, weighing private criteria:

$$\mathrm{F} = q_1\mathrm{P} + q_2\mathrm{T} \Rightarrow \min. \quad (3)$$

But coefficients q_1, q_2 depend on the situation and values P and T. If, for example, a person wanted to save one day and increased weight coefficient q_1 at P, and, at the same time, was late for work and received a reprimand, on another day he will increase weight coefficient q_2 at T. With an arbitrary assignment of weights q_1, q_2, there is no guarantee to get an ‘optimal’ solution.

A more complex expression is often used:

$$\mathrm{F} = \frac{q_1F_1 + q_2F_2 + \ldots}{s_1Q_1 + s_2Q_2 + \ldots}, \quad (4)$$

but this formula also has the disadvantages inherent in the previous methods.

In this case we encounter a typical technique – there is a transfer of arbitrariness from one method to another. Simply choosing a compromise solution, analyzing and comparing all the pros and cons of each alternative, seems not scientific enough and too arbitrary. But the use of a formula including coefficients q_1, q_2, …, s_1, s_2, … is already considered ‘science’! In fact, there is no science here, and you do not have to deceive yourself. It is impossible to completely get rid of subjectivity in multi-criteria tasks.

There are rare situations when the analysis of the criteria allows you to unambiguously choose the option that is best than the others by all odds. But in managerial tasks of choosing a solution is not usually obvious: with an improvement of one indicator, another one worsens.

Thus, it is generally impossible to get an optimal solution to a multi-criteria problem, and the mathematical methods only help to discard the admittedly inferior solution options, which are, in all respects, worse than others, and choose a compromise option from the remaining ones.

2. Pareto Optimality

Suppose there is a decision-making task with n criteria F_1, F_2, …, F_n and suppose it is necessary to maximize these criteria. We will make a pair comparison of possible alternatives. If for any pair of solutions y_1 and y_2 the values of indicators F_1, F_2, …, F_n for $y_1 \geq$ the values of the corresponding indicators for y_2, and at least one value of the indicator is strictly larger, then solution y_1 is better than solution y_2, in which case it is said that y_1 ‘dominates’ y_2. Then solution y_2 can be discarded but y_1 remains. Thus, after such a pairwise comparison and selection for further analysis, only *‘Pareto-optimal’* solutions will remain over which there are no dominant solutions (Italian sociologist and economist V. Pareto, 1848–1923).

Consider an example of finding Paretan solutions. There is a task with criteria F_1 and F_2 that you would like to maximize. We denote possible solutions with $x_1, x_2, \ldots, x_k$. We shall calculate values F_1, F_2 for each solution and put points with these coordinates on the Cartesian plane. Let us number the points according to the solution number (Fig. 1).

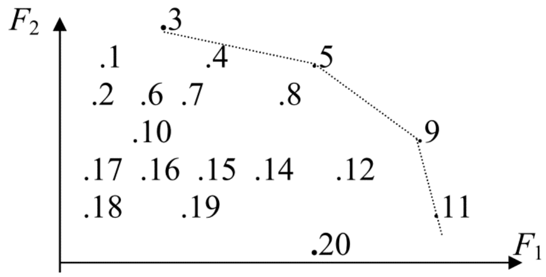

Fig. 1. Pareto optimality

Making a pairwise comparison of the solutions in Fig. 1, we see that x_3 *dominates* x_1; x_5 *dominates* x_4, x_8; x_9 *dominates* x_{12}, x_{20}, etc. Solutions x_3, x_5, x_9, x_{11} do not have dominant solutions, therefore they are Pareto-optimal. Such a selection significantly reduced the number of solutions, and a decision maker should choose a preferred option from them.

With three or more criteria, geometric interpretation is impossible, but the procedure for selecting Paretan solutions is similar – using the method of pairwise comparisons.

Let us consider an example of Finnish budgeting. The following criteria were used to select the solution:

F_1 – increase in gross national product (GNP), %;

F_2 – decrease in unemployment, %;

F_3 – decrease in inflation, %;

F_4 – decrease in trade deficit (DM billion).

The criteria values for the various budget options are shown in Table 1.

Table 1

Solution options and criteria values

Solution options	F_1	F_2	F_3	F_4
1	–2.76	3.27	8.14	2.23
2	0.54	2.83	9.06	5.26
3	1.85	2.65	8.84	6.55
Best criteria values	7.14	1.87	8.14	1.23

The optimal criteria values, when finding a solution and taking into account only one criterion, are given in the last line of Table 1. At the same time, these values are not achieved according to all the criteria. The options given in Table 1 are Pareto-optimal solutions in a task with four criteria, with none of them dominating the others. Thus, option 1 has the best indicator F_4 (trade deficit and inflation), but the worst in terms of unemployment and GNP growth. Option 3 is the best in terms of GNP growth and unemployment, but the worst in terms of trade deficit. Such contradictions are typical for multi-criteria tasks.

The choice of the final solution remains the prerogative of a person who, by virtue of his experience and qualifications, can take responsibility and make an *acceptable* compromise decision.

3. Dialog solution method

The decision selection process can take place in *a dialog (interactive)* mode, wherein the computer outputs the values of criteria $F_1, F_2, \ldots, F_n$, and the *decision maker*, having analyzed the information, changes the parameters of the criteria calculations and repeats the calculations until a compromise solution is obtained.

While applying the dialog method, a multi-criteria problem is often reduced to a single-criterion one by selecting and optimizing one main criterion F_1, while all the other criteria are limited to some acceptable values. For example, in strategic planning of the development of the region, you can minimize costs, while ensuring a given rate of growth in average wages, the number of jobs, the level of environmental safety, etc. With this method, all the criteria except the main one (costs) follow the specified restrictions. You can make adjustments to these constraints in a dialog mode.

4. Sequential concession method

In the dialog mode, *the sequential concession method* can be implemented. This method is used if criteria F_1, F_2, …, F_n can be ordered by their importance. The first step is a solution that optimizes the first most important criterion $F_1 = F_1^*$. At the second stage, some 'concession' is made; F_1 is changed by ΔF_1 and transformed to the restriction, after which optimization according to the second criterion F_2 is carried out. Further ΔF_2 'concession' by criterion F_2 is similarly made, transition to optimization by F_3 is carried out and etc. This method allows you to immediately see in the process of solving a multi-criteria task which 'concession' according to one criterion makes another one win.

Once again, the choice of a solution of a multi-criteria task is not determined unambiguously. The decision maker, by analyzing the relevant data provided to him on the advantages and disadvantages of possible alternatives, makes the choice consciously, taking into account the specific situation and personal preferences.

5. Hierarchy Analysis Method

The point of this method is to represent a multi-criteria task in the form of a multi-level hierarchical structure consisting of criteria and alternatives. It is proposed by T. Saaty [11] and its distinctive feature is simplicity of examination of pairwise comparison of criteria and alternatives by the degree of preference. Priority vectors of relative importance of criteria against each other and relative importance of alternatives for each criterion are formed.

At the top level of the hierarchy there is the goal of solving the task, then come the criteria and alternatives.

The elements of each level are then compared in pairs. The dominance of the elements over each other is assessed by a nine-point scale (Table 2).

Table 2

Element importance ratio scale

Degree of importance a_{ij}	Preferences
1	The elements are equivalent
3	The element has some preference over the other
5	The element has a significant preference over the other
7	The element has a very strong preference over the other
9	The element has an absolute preference over the other
2, 4, 6, 8	Intermediate values
Reciprocal values $a_{ji} = 1/a_{ij}$,	Element j, when compared to element i, has the reciprocal value

Assessments are carried out by experts using various methods: the arithmetic mean method, the median method, the Kemeny method, etc. The combination of methods produces a more objective result. There is a variety of reading matter on the application of expert methods [16–21].

As a result of this examination, square matrices of paired comparisons of dimension n are formed for each level of hierarchy $A = \{a_{ij}\}$, where n is the number of elements compared in pairs.

The eigenvectors of each pair comparison matrix (W^A) define priority vectors. They are calculated as follows:

– the matrix is normalized, with all its elements divided by the sum of the elements of the corresponding column. Then, the mean arithmetic values of the elements of each row of the normalized matrix, which are components of vector W^A, are calculated.

Each matrix A must be checked for the validity of expert opinions, for which *the maximum eigenvalue* λ_{max} is found according to the following algorithm:

– matrix A is multiplied on the right by its own vector W^A, after which all the components of the formed vector are added.

The found value λ_{max} is used to calculate the consistency index using the formula:

$$CI = (l_{max} - n)/(n-1). \quad (5)$$

The consistency index assesses the inconsistency of expert opinions when comparing the criteria and alternatives. Contradictions are associated with subjective errors of experts. A small number of inconsistencies correspond to a small consistency index value.

Then the consistency ratio CR = CI/CC is calculated, where CC is the mathematical expectation (average value) of the consistency index of a randomly obtained matrix of pairwise comparisons. CC is approximately calculated by the following formula:

$$\mathrm{CC}=\frac{1.98(n-2)}{n}. \qquad (6)$$

CR value should be ≤ 0.1, at least ≤ 0.15. Otherwise, the provided judgments of the experts should be rechecked.

Priority vectors are sequentially defined from the lower levels of the hierarchy to the upper levels.

Example. Let us consider the problem of choosing a project when developing and implementing a program for running a national project in the federal subject from three proposed options according to four criteria: Population Welfare, Ecological Condition, Number of Jobs and Cost. Let us construct a three-level structure of the alternatives and criteria, as shown in Fig. 2. The matrix of pairwise comparisons of criteria is given in Table 3.

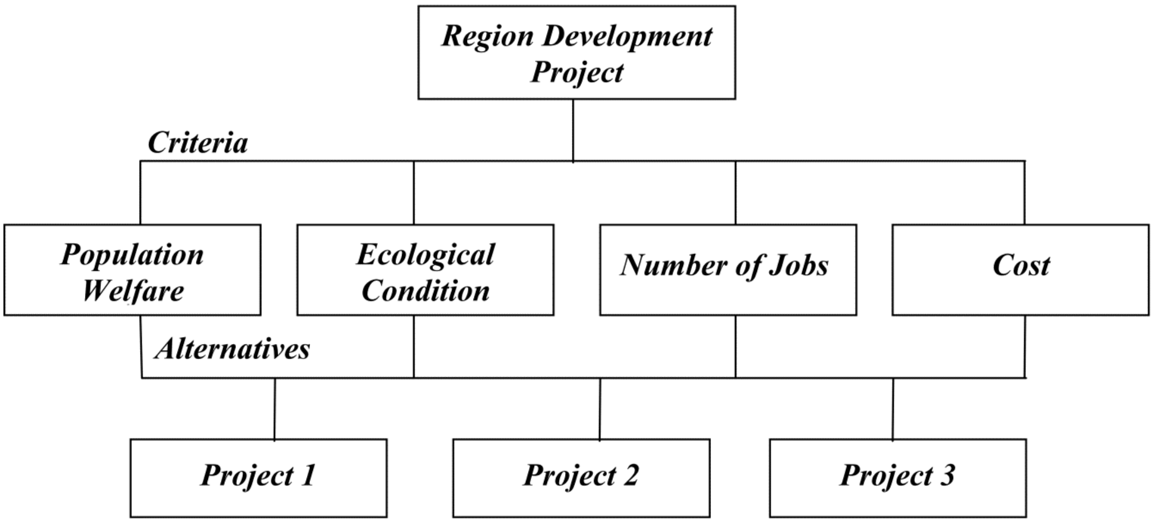

Fig. 2. Structure of alternatives and criteria

Table 3

Matrix of pairwise comparisons of criteria (A)

Region Development Project	Population Welfare	Ecological Condition	Number of Jobs	Cost
Population Welfare	1	5	2	1/3
Ecological Condition	1/5	1	1/3	1/4
Number of Jobs	1/2	3	1	1/6
Cost	3	4	6	1

Let us calculate λ_{max} by formula (5).

To do this, first let us normalize matrix A:

$$A=\begin{pmatrix} 1 & 5 & 2 & 1/3 \\ 1/5 & 1 & 1/3 & 1/4 \\ 1/2 & 3 & 1 & 1/6 \\ 3 & 4 & 6 & 1 \end{pmatrix}; \quad N_A=\begin{pmatrix} 0.21 & 0.38 & 0.21 & 0.19 \\ 0.04 & 0.07 & 0.04 & 0.14 \\ 0.11 & 0.23 & 0.11 & 0.1 \\ 0.64 & 0.31 & 0.64 & 0.57 \end{pmatrix}.$$

Let us then calculate the components of eigenvector W^A.

$w_{\text{Population Welfare}} = (0.21 + 0.38 + 0.21 + 0.19)/4 = 0.2475;$

$w_{\text{Ecological Condition}} = (0.04 + 0.07 + 0.04 + 0.14)/4 = 0.0725;$

$w_{\text{Number of Jobs}} = (0.11 + 0.23 + 0.11 + 0.1)/4 = 0.1375;$

$w_{\text{Cos}t} = (0.64 + 0.31 + 0.64 + 0.57)/4 = 0.54.$

$\overline{w_1} = 0.2475;\ \overline{w_2} = 0.0725;\ \overline{w_3} = 0.1375;\ \overline{w_4} = 0.54.$

$$[A]\,W^A = \begin{pmatrix} 1 & 5 & 2 & 0.33 \\ 0.2 & 1 & 0.33 & 0.25 \\ 0.5 & 3 & 1 & 0.17 \\ 3 & 4 & 6 & 1 \end{pmatrix} \begin{pmatrix} 0.2475 \\ 0.0725 \\ 0.1375 \\ 0.54 \end{pmatrix} = \begin{pmatrix} 1.0632 \\ 0.302 \\ 0.5705 \\ 2.3975 \end{pmatrix}.$$

Hence:

$\lambda_{\max} = 1.0632 + 0.302 + 0.5705 + 2.3975 = 4.33333.$

Let us investigate the consistency of the matrix.

Consistency Index:

$$\text{CI} = \frac{\lambda_{\max} - n}{n-1} = \frac{4.3 - 4}{4 - 1} = 0.1.$$

Consistency ratio:

$$\text{CC} = \frac{1.98(4-2)}{4} = 0.99;$$

$$\text{CR} = \frac{\text{CI}}{\text{CC}} = \frac{0.1}{0.99} = 0.1.$$

$\text{CI} \le 0.1$, therefore, the consistency level of matrix A is acceptable.

Matrix of pairwise comparisons of alternatives according to Population Welfare is given in Table 4.

Table 4

Matrix of pairwise comparisons of alternatives according to Population Welfare (criterion P)

Population Welfare	Project 1	Project 2	Project 3
Project 1	1	5	3
Project 2	1/5	1	1/2
Project 3	1/3	2	1

Let us normalize matrix P:

$$\text{P} = \begin{pmatrix} 1 & 5 & 3 \\ 1/5 & 1 & 1/2 \\ 1/3 & 2 & 1 \end{pmatrix};\quad \text{N}_\text{P} = \begin{pmatrix} 0.65 & 0.625 & 0.67 \\ 0.13 & 0.125 & 0.11 \\ 0.22 & 0.25 & 0.22 \end{pmatrix}.$$

$w_{\text{project1}} = (0.65 + 0.625 + 0.67)/3 = 0.648;$

$w_{\text{project2}} = (0.13 + 0.125 + 0.11)/3 = 0.121;$

$w_{\text{project3}} = (0.22 + 0.25 + 0.22)/3 = 0.23.$

$\overline{w_1} = 0.648;\ \overline{w_2} = 0.121;\ \overline{w_3} = 0.23.$

$$[P]\,W^P = \begin{pmatrix} 1 & 5 & 3 \\ 0.2 & 1 & 0.5 \\ 0.33 & 2 & 1 \end{pmatrix} \begin{pmatrix} 0.648 \\ 0.121 \\ 0.23 \end{pmatrix} = \begin{pmatrix} 1.943 \\ 0.3656 \\ 0.6858 \end{pmatrix}.$$

$\lambda_{\max} = 1.943 + 0.3656 + 0.6858 = 2.9944 \approx 3.$

Let us investigate the consistency of the matrix:

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n-1} = \frac{3-3}{2} = 0;$$

$$\mathrm{CC} = \frac{1.98(3-2)}{3} = 0.66;$$

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{CC}} = 0.$$

Matrix of pairwise comparisons of alternatives according to Ecological Condition is given in Table 5.

Table 5

Matrix of pairwise comparisons of alternatives according to Ecological Condition (criterion J)

Ecological Condition	Project 1	Project 2	Project 3
Project 1	1	4	8
Project 2	1/4	1	6
Project 3	1/8	1/6	1

$$J = \begin{pmatrix} 1 & 4 & 8 \\ 0.25 & 1 & 6 \\ 0.125 & 0.17 & 1 \end{pmatrix}; \quad \mathrm{N_J} = \begin{pmatrix} 0.72 & 0.77 & 0.53 \\ 0.18 & 0.19 & 0.4 \\ 0.1 & 0.03 & 0.07 \end{pmatrix}.$$

$$w_{\text{project1}} = (0.72 + 0.77 + 0.53)/3 = 0.67;$$

$$w_{\text{project2}} = (0.18 + 0.19 + 0.4)/3 = 0.25;$$

$$w_{\text{project3}} = (0.1 + 0.03 + 0.07)/3 = 0.03.$$

$$\overline{w_1} = 0.67; \ \overline{w_2} = 0.25; \ \overline{w_3} = \ 0,03.$$

$$[\mathrm{J}]\,\mathrm{W}^{\mathrm{J}} = \begin{pmatrix} 1 & 4 & 8 \\ 0.25 & 1 & 6 \\ 0.125 & 0.17 & 1 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.25 \\ 0.03 \end{pmatrix} = \begin{pmatrix} 1.910 \\ 0\ .5975 \\ 0.1565 \end{pmatrix}.$$

$$\lambda_{\max} = 1.910 + 0.5975 + 0.1565 = 2.6637.$$

Let us investigate the consistency of the matrix:

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n-1} = \frac{2.67-3}{2} = -0.165;$$

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{CC}} = \frac{-0.165}{0.66} = -0.25.$$

Matrix of pairwise comparisons of alternatives according to Number of Jobs is given in Table 6.

Table 6

Matrix of pairwise comparisons of alternatives according to Number of Jobs (criterion N)

Number of Jobs	Project 1	Project 2	Project 3
Project 1	1	6	1/3
Project 2	1/6	1	1/4
Project 3	3	4	1

$$\mathrm{N} = \begin{pmatrix} 1 & 6 & 0.33 \\ 0.17 & 1 & 0.25 \\ 3 & 4 & 1 \end{pmatrix}; \quad \mathrm{N_N} = \begin{pmatrix} 0.24 & 0.54 & 0.21 \\ 0.04 & 0.09 & 0.15 \\ 0.72 & 0.4 & 0.63 \end{pmatrix}.$$

$w_{\text{project1}} = (0.24 + 0.54 + 0.21) / 3 = 0.33;$

$w_{\text{project2}} = (0.04 + 0.08 + 0.15) / 3 = 0.09;$

$w_{\text{project3}} = (0.72 + 0.4 + 0.63) / 3 = 0.4375.$

$\overline{w_1} = 0.33;\ \overline{w_2} = 0.09;\ \overline{w_3} = 0.4375.$

$$[\text{N}]\,\text{W}^{\text{N}} = \begin{pmatrix} 1 & 6 & 0.33 \\ 0.17 & 1 & 0.25 \\ 3 & 4 & 1 \end{pmatrix} \begin{pmatrix} 0.33 \\ 0.09 \\ 0.44 \end{pmatrix} = \begin{pmatrix} 1.0152 \\ 0.2561 \\ 1.79 \end{pmatrix}.$$

$\lambda_{\max} = 1.0152 + 0.2561 + 1.79 = 3.0613.$

Let us investigate the consistency of the matrix:

$$\text{CI} = \frac{\lambda_{\max} - n}{n-1} = \frac{3.0613 - 3}{2} = 0.03;$$

$$\text{CR} = \frac{\text{CI}}{\text{CC}} = \frac{0.03}{0.66} = 0.05.$$

Matrix of pairwise comparisons of alternatives according to Cost is given in Table 7.

Table 7

Matrix of pairwise comparisons of alternatives according to Cost (criterion C)

Cost	Project 1	Project 2	Project 3
Project 1	1	4	7
Project 2	1/4	1	5
Project 3	1/7	1/5	1

$$\text{C} = \begin{pmatrix} 1 & 4 & 7 \\ 0.25 & 1 & 5 \\ 0.14 & 0.2 & 1 \end{pmatrix};\quad \text{N}_{\text{C}} = \begin{pmatrix} 0.72 & 0.76 & 0.54 \\ 0.18 & 0.19 & 0.39 \\ 0.1 & 0.04 & 0.08 \end{pmatrix}.$$

$w_{\text{project1}} = (0.72 + 0.76 + 0.54)/3 = 0.67;$

$w_{\text{project2}} = (0.18 + 0.19 + 0.39)/3 = 0.25;$

$w_{\text{project3}} = (0.1 + 0.04 + 0.08)/3 = 0.07.$

$\overline{w_1} = 0.67;\ \overline{w_2} = 0.25;\ \overline{w_3} = 0.07.$

$$[\text{C}]\,\text{W}^{\text{C}} = \begin{pmatrix} 1 & 4 & 7 \\ 0.25 & 1 & 5 \\ 0.14 & 0.2 & 1 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.25 \\ 0.07 \end{pmatrix} = \begin{pmatrix} 2.16 \\ 0.7576 \\ 0.2138 \end{pmatrix}.$$

$\lambda_{\max} = 2.16 + 0.7576 + 0.2138 = 3.1314.$

Let us investigate the consistency of the matrix:

$$\text{CI} = \frac{\lambda_{\max} - n}{n-1} = \frac{3.1314 - 3}{2} = 0.065;$$

$$\text{CR} = \frac{\text{CI}}{\text{CC}} = \frac{0.065}{0.66} = 0.09.$$

All the generated priority vectors are put into Table 8. The resultant vector is determined by multiplying the matrix composed of priority vectors of alternatives by a priority vector of criteria.

Table 8

The generated priority vectors

Region Development Project	Population Welfare	Ecological Condition	Number of Jobs	Cost	Resultant Vector
Criteria	0.2475	0.0725	0.1375	0.54	–
Project 1	0.648	0.67	0.33	0.67	0.61613
Project 2	0.121	0.25	0.09	0.25	0.1954
Project 3	0.23	0.03	0.4375	0.07	0.1571

Project 1: 0.2475 · 0.648 + 0.0725 · 0.67 + 0.1375 · 0.33 + 0.54 · 0.67 = **0.61613**.

Project 2: 0.2475 · 0.121 + 0.0725 · 0.25 + 0.1375 · 0.09 + 0.54 · 0.25 = 0.1954.

Project 3: 0.2475 · 0.23 + 0.0725 · 0.03 + 0.1375 · 0.4375 + 0.54 · 0.07 = 0.1571.

Project 1 is the best option when choosing a region development project.

The method of analyzing hierarchies can be used in various areas of political science for tasks related to forecasting electoral behavior, turnout, campaign costs, etc. Taking into account the influence of the unstable external environment, these tasks can highlight key areas of improving the quality of decisions made and make an expert assessment of their significance taking into account the specifics of interests and influences of the subjects of the analyzed political system.

Conclusion

The article shows that various multicriteria optimization methods provide many effective solutions to management problems. This leads to the need to apply and analyze the totality of the considered methods to select the best solution that can be used to improve political processes.The use of mathematical modeling and methods to solve multi-criteria management problems helps politicians make efficient decisions in their work and provides them with communication tools by using professional mathematical language.

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Contribution of the authors: the authors contributed equally to this article.

The authors declare no conflicts of interests.

Вклад авторов: все авторы сделали эквивалентный вклад в подготовку публикации.

Авторы заявляют об отсутствии конфликта интересов.

The article was submitted 28.12.2023

Статья поступила в редакцию 28.12.2023