Управление в социально-экономических системах

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PRINCIPLE OF COORDINATED PLANNING IN CONTROL OF DISTRIBUTED PROJECTS AND PROGRAMS

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The paper deals with the problem of managing distributed projects and programs. These programs consist of subprograms distributed functionally, administratively or geographically. For instance, a program of regional development includes a subprogram of environmental safety. In this regard, the main problem of managing distributed programs is the problem of interests' coordination for all persons concerned. We propose the principle of coordinated planning for designing implementation plans of distributed programs.

Keywords: distributed programs, environmental safety, the principle of coordinated planning.

1. Introduction

Distributed projects (programs) are projects (programs) consisting of subprojects (subprograms) distributed functionally, administratively or geographically. Functional distribution means that there exist different functional directions of a project (program) having a dedicated subproject (subprogram) with a separate manager and team. Among examples, we mention a regional development program which includes several functional directions such as social development, economic development, environmental safety and others. In the case of administrative distribution, there are subprojects (subprograms) in the interests of different administrative or economic institutions. For instance, a regional development program includes development subprograms of member cities, municipalities, etc. with separate managers and teams. The main feature of functionally and administratively distributed projects (programs) is the presence of noncoinciding interests pursued by the managers of associated subprojects (subprograms). Therefore, the major problem in managing functionally and administratively distributed projects (programs) lies in interests' coordination for all persons concerned (basically, the managers of subprojects and subprograms). Geographically distributed projects (programs) can be functionally and administratively distributed and, moreover, have another essential peculiarity. While designing implementation plans of such projects (programs), one should take into account the transfer time of different resources (personnel, equipment, materials): this time is often comparable with (or even exceeds!) the execution time of a job. The reparation and construction of motor roads, railway tracks and bridges are the examples of such projects.

2. The principle of coordinated planning in distributed projects (programs)[1]

We study the problem of interests' coordination among the sub-projects (subprograms) of a functionally or administratively distributed project (program) using the example of a functionally distributed program. All results can be easily applied to geographically or administratively distributed projects and programs.

Consider a functionally distributed program composed of m subprograms covering different directions. In the sequel, the program manager will be called the Principal (P), whereas subprogram managers will be called agents (A).

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[1] This section was written together with Chu Dong Xuan, a postgraduate student at Voronezh State University of Architecture and Civil Engineering (Voronezh, Russia).

Suppose that there is a state assessment of each direction (in a qualitative or quantitative scale). Denote by F_i, the state assessment of direction i (the goal function of agent i) and by F the goal function of the Principal. The goal function of the Principal depends on the goal functions of agents:

$$F = \Phi(F_1; F_1; \dots; F_m). \tag{1}$$

This can be a linear, additive or matrix convolution.

The Principal has to design a program (a set of projects) in order to maximize the goal function F under limited resources R allocated to the program. On the other hand, each agent / strives to design a subprogram maximizing its goal function F_i.

Imagine that the Principal ignores the interests of agents during program design. This would cause a series of negative consequences such as hiding or misrepresentation of information provided by agents to the Principal, non-fulfillment of program measures, etc. For interests' coordination between the Principal and agents, the theory of active systems proposes the principle of coordinated planning [1]. The fundamental idea of this principle is to optimize the Principal's goal function on the set of coordinated plans (i.e., plans such that the goal functions of agents are not smaller than a given threshold). For formal statement of the coordinated planning problem, designate by F_i^0 the existing state assessment of direction i. The interests' coordination condition can be a guaranteed increment $\Delta F_i = \gamma_i F_i^0$ of the function F_i, (i.e., the increase by $100\gamma_i$ percent). In this case, the coordinated planning problem acquires the form

$$F = \Phi(F_1; F_1; \dots; F_m) \to \max \tag{2}$$

subject to the constraints

$$F_i \ge (1+\gamma_i) F_i^0, \quad i = \overline{1,n}. \tag{3}$$

3. Problem statement

There are n candidate projects for inclusion in the program. Implementation of each project i incurs the costs c_i and yields the economic effect a_{ij} for direction j (we comprehend effect as the increment of the goal function F_j). Set $x_i = 1$ if project i is included in the program ($x_i = 0$, otherwise).

The problem. Find $x = \{x_i, i = \overline{1,n}\}$ maximizing the functions

$$\Phi(y_1, y_2, \dots, y_m), \text{ where } y_j = \sum_i a_{ij} x_i, \ j = \overline{1,m} \tag{4}$$

subject to the constraints

$$\sum_i c_i x_i \le R, \tag{5}$$

$$\sum_i x_i a_{ij} \ge \gamma_j F_j^0, \ j = \overline{1,m}. \tag{6}$$

3.1. SPECIAL CASE. SINGLE-PURPOSE PROJECTS

Consider the following special case of the problem. For each direction j, there exists a set Q_j of projects contributing to it; the sets Q_j do not intersect. In this case, the problem is treated in two stages.

Stage 1. Solve m knapsack problems: maximize

$$y_j = \sum_{i \in Q_j} a_i x_i \tag{7}$$

subject to the constraints

$$\sum_{i \in Q_J} x_i c_i \le R_j, \tag{8}$$

$$\sum_{i \in Q_j} x_i a_i \ge \gamma_j F_j^0 = b_j, \text{ where } 0 \le R_j \le R. \tag{9}$$

For this, solve the standard knapsack problem (7), (8) under $R_j = R$.

As is well-known, solution of the knapsack problem under $R_j = R$ yields optimal solutions for all $R_j < R$. Denote by $Y_j(R_j)$ the value of the goal function (7) in the optimal solution as a function of R_j. Find the minimum value $R_j = d_j$ such that $Y_j(d_j) \ge b_j$. As a result, we obtain a relationship $Y_j(R_j)$, where $d_j \le R_j \le R$.

Stage 2. Solve the maximization problem of the function

$$Y(R) = \sum_j Y_j(R_j) \qquad (10)$$

subject to the constraints $R_j \geq b_j$, $j = \overline{1,m}$,

$$\sum_{j=1}^{m} R_j \leq R. \qquad (11)$$

Here we apply dichotomous programming. Each knapsack problem is solved by the backward method.

Example 1. Consider three directions of a program, see Table 1.

Table 1

j	Direction 1				Direction 2				Direction 3			
i	1	2	3	4	5	6	7	8	9	10	11	12
a_i	12	30	50	16	16	15	8	18	24	18	10	7
c_i	6	5	10	4	4	3	4	3	12	6	5	7

Set $b_1 = 20$, $b_2 = 34$, $b_3 = 20$, $R = 30$.

Stage 1. Solve the knapsack problem for direction 1 using dichotomous programming [2]. Fig. 1 shows the dichotomous representation tree of this problem.

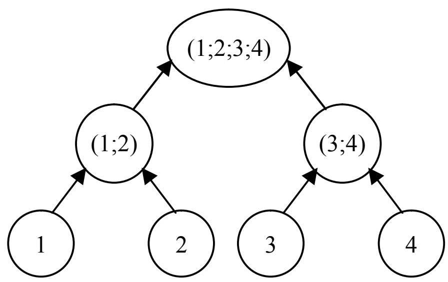

Fig. 1

Step 1. Solve the problem for projects 1 and 2. The resulting solution is described by Table 2. Here the first value indicates the costs and the second value means the economic effect.

Table 2

1	5;30	11;42
0	0;0	6;12
2 / 1	0	1

The results are combined in Table 3. Actually, this table contains only Pareto optimal variants. For instance, we eliminate variant (6;12) as being dominated by variant (5;30) (under smaller costs, it yields higher effect).

Table 3

Variant	0	1	2
Costs	0	5	11
Effect	0	30	42

Step 2. Solve the problem for projects 3 and 4. The solution is illustrated by Table 4. The results can be found in Table 5.

Table 4

1	4;16	14;66
0	0;0	10;50
4 / 3	0	1

Table 5

Variant	0	1	2	3
Costs	0	4	10	14
Effect	0	16	50	66

Step 3. Consider the united projects (1;2) and (3;4). The solution is provided by Table 6 and the results are combined in Table 7. As far as $b_1 = 20$, we reject variants (0;0) and (4;16).

Table 6

2	11;42	15;58	21;92	25;108
1	5;30	9;46	15;80	19;96
0	0;0	4;16	10;50	14;66
(1;2) / (3;4)	0	1	2	3

Table 7

Variant	1	2	3	4	5	6	7
R_1	5	9	10	14	15	19	25
Y_1	30	46	50	66	80	96	108

Solve the knapsack problem for direction 2. The solution is described by Table 8.

Table 8

Variant	1	2	3
R_2	7	10	14
Y_2	34	49	57

Solve the knapsack problem for direction 3. The solution is described by Table 9

Table 9

Variant	1	2	3	4	5
R_3	11	17	18	23	30
Y_3	28	34	42	52	59

Stage 2. Solve the optimization problem

$$Y_1(R_1) + Y_2(R_2) + Y_3(R_3) \to \max \quad (12)$$

to the constraint

$$R_1 + R_2 + R_3 \le 30. \quad (13)$$

Step 1. Consider directions 1 and 2. The solution is provided by Table 10. And the results can be found in Table 11.

Table 10

14;57	19;87	23;103	24;107	28;123	29;137	–	–
10;49	15;79	19;95	20;99	24;115	25;129	29;142	–
7;34	12;64	16;80	17;84	21;100	22;114	26;130	–
2 / 1	5;30	9;46	10;50	14;66	15;80	19;96	25;108

Table 11

Variant	1	2	3	4	5	6	7	8	9	10	11	12
$R_1 + R_2$	12	15	16	17	19	20	21	22	24	25	26	29
$Y_1 + Y_2$	64	79	80	84	95	99	100	114	115	129	130	142

Step 2. Consider united direction (1;2) and direction 3. The solution is combined in Table 12.

Table 12

19;95	0;123	–	–
17;84	28;112	–	–
16;80	27;108	–	–
15;79	26;107	–	–
12;64	23;92	29;98	30;106
(1;2) / (3)	11;28	17;34	18;42

In Table 12 find a cell with the maximum second value. This is cell (30;123) associated with the effect 123. Cell (30;123) corresponds to variant 5 in Table 11 and variant 1 in Table 9. This variant corresponds to the solution of the knapsack problem

$x_9 = 0; x_{10} = 1; x_{11} = 1; x_{12} = 0$

with costs 11 and effect 28.

Variant 5 in Table 11 corresponds to cell (19;95) in Table 10, i.e., variant 2 in Table 8 and variant 2 in Table 7.

Next, variant 2 in Table 8 corresponds to the solution of the knapsack problem for direction 2:

$x_5 = 1; x_6 = 1; x_7 = 0; x_8 = 1$

with costs 10 and effect 49.

And finally, variant 2 in Table 7 corresponds to the following solution of the knapsack problem for direction 1:

$x_1 = 0; x_2 = 1; x_3 = 0; x_4 = 1$

with costs 9 and effect 46.

3.2. GENERAL CASE. MULTI-PURPOSE PROJECTS

In the general case, there exist projects whose implementation contributes to several directions. Such projects are said to be multipurpose projects. If the number q of multi-purpose projects is not large, consider all 21' variants of multi-purpose projects inclusion in the program and choose the best one (perform exhaustive search).

Example 2. Take 2 directions and 8 projects with the parameters described by Table 13.

Table 13

i	1	2	3	4	5	6	7	8
a_{i_1}	12	18	15	24	15			

Clearly, projects 4 and 5 are multi-purpose. Set $b_1 = 20$, $b_2 = 25$, $R = 30$.

Variant 1. None of the multi-purpose projects is included in the program, i.e., $x_4 = x_5 = 0$.

Stage 1. Solve the problem for direction 1: maximize

$12x_1 + 18x_2 + 15x_3$

subject to the constraint

$4x_1 + 9x_2 + 3x_3 \le R_1$,

where $R_1 < 30$. The solution is illustrated by Table 14.

Table 14

Variant	0	1	2	3	4
R_1	0	3	7	12	16
Y_1	0	15	37	33	45

Solve the problem for direction 2: maximize

$16x_6 + 21x_7 + 24x_8$

subject to the constraint

$4x_6 + 7x_7 + 12x_8 \le R_2$,

where $R_2 < 30$. The solution is defined by Table 15.

Table 15

Variant	0	1	2	3	4	5	6
R_2	0	4	7	11	16	19	23
Y_2	0	16	21	37	40	45	61

Stage 2. Maximize

$Y_1(R_1) + Y_2(R_2)$

subject to the constraint

$R_1 + R_2 \le 30$.

The solution can be found in Table 16.

Table 16

4	16;45	20;61	23;66	27;82	–	–	–
3	12;33	16;49	19;54	23;70	28;73	–	–
2	7;27	11;43	14;48	18;64	23;67	26:72	30;88
1	3;15	7;31	10;36	14;52	19;55	22;60	26;76
0	0;0	4;16	7;21	11;37	16;40	19;45	23;61
1 / 2	0	1	2	3	4	5	6

As far as $b_1 = 20$, eliminate rows 0 and 1 from Table 16. Similarly, eliminate columns 0, 1 and 2 due to $b_2 = 25$. In the resulting table, identify a cell with the maximum second value. Actually, this is cell (30;88) associated with effect 88.

Variant 2. Project is included in the program ($x_4 = 1$; $x_5 = 0$). In this case, the residual resource makes up $R' = 30 - 8 = 22$. So long as $a_{41} = 24$ and $a_{42} = 16$, then $b_1' = 0$ and $b_2' = 25 - 16 = 9$. Hence, we have to eliminate only column 0 and row 0 from Table 16.

Define a cell with the maximum second value among all cells whose first value does not exceed 22. This is cell (18;64) with effect 64. By adding the effect from project 4 ($a_{41} + a_{42} = 40$), we get total effect 104.

Variant 3. Project 5 is included in the program. Hence,

$R' = 30 - 10 = 20$, $b_1' = 20 - 15 = 5$, $b_2' = 25 - 10 = 15$.

Similarly, to the previous variant, eliminate column 0 and row 0 from Table 16. Find a cell with the maximum second value among all cells whose first value does not exceed 20. This is cell (18; 64) yielding effect 64. By adding the effect from project 5, we get total effect 64 + 25 = 89.

Variant 4. Projects 4 and 5 are included in the program ($x_4 = x_5 = 1$). Then we have that $R' = 30 - 18 = 12$, $b_1' = 0$, and $b_2' = 0$. Identify a cell with the maximum second value among all cells whose first value does not exceed 12. This is cell (11;43) with effect 43. By adding the effects from projects 4 and 5, we get total effect 43 + 40 + 25 = 108. The maximum effect is gained by variant 4. Note that cell (11;43) corresponds to variant 1 in Table 15 and variant 2 in Table 14. On the other hand, variant 1 in Table 15 corresponds to the following solution for direction 2:

$x_6 = 1, x_7 = 0, x_8 = 0.$

Variant 2 in Table 14 corresponds to the following solution for direction 1:

$x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 0.$

And finally, we establish that the program includes projects 1, 3, 4, 5 and 6 with total effect 108 and total costs 29.

3.3. NETWORK PROGRAMMING METHOD

Under a large number of multi-purpose projects, program design based on their exhaustive search becomes inefficient. Consider the branch-and-bound method with estimation using network programming [3]. Let us illustrate this method for the inverse problem: minimize the costs required for obtaining a given total effect. In other words, the problem is to minimize the goal function

$$C(x) = \sum_i c_i x_i$$

subject to the constraint

$$\sum_j y_j \ge B,$$

$$y_j \ge b_j,\ j = \overline{1,m}.$$

We provide a simple example below.

Example 3. There are 4 projects with the parameters described by Table 17. The number of directions equals 2.

Table 17

i	1	2	3	4
a_{i_1}	12	6	9	
a_{i_2}		4	6	8
c_i	3	2	4	3

Set $b_1 = 10$, $b_2 = 8$ and $B = 30$. According to Table 17, projects 2 and 3 are multi-purpose. Fig. 2 shows the network representation of the associated constraints.

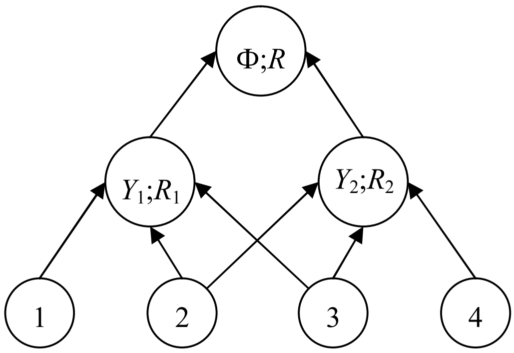

Fig. 2

Theory of network programming prescribes splitting arbitrarily the costs c_2 and c_3 of multi-purpose projects into two components s_{21}, s_{22} and s_{31}, s_{32}, respectively (since vertices 2 and 3 have 2 outgoing arcs, see Fig. 2). For instance, take $s_{21} = s_{22} = 1$, $s_{31} = 1$, $s_{32} = 3$. This leads to two estimation problems for each direction. The estimation problem for direction 1: minimize

$$C_1(x) = 3 \times x_1 + 1 \times x_2 + 1 \times x_3$$

subject to the constraint

$$12x_1 + 6x_2 + 9x_3 \ge B_1$$

where $d_1 \le B_1 \le B$.

Denote by $Z_1(B_1)$ the optimal value of $C_1(x)$. The solution is described by Table 18.

Table 18

Variant	0	1	2	3	4
Z_1	0	1	2	4	5
B_1	0	6	15	21	27

We eliminate variants 0 and 1, since $B_1 < b_1 = 10$ for them.

The estimation problem for direction 2 has the form

$$C_2(x) = 1 \times x_2 + 3 \times x_3 + 3 \times x_4 \to \min$$

subject to the constraint

$$4x_2 + 6x_3 + 8x_4 \geq B_2$$

where $d_2 \leq B_2 \leq B$.

Designate by $Z_2(B_2)$ the optimal value of $C_2(x)$. The solution is defined by Table 19.

Table 19

Variant	0	1	2	3	4	5
Z_2	0	1	3	4	6	7
B_2	0	4	8	12	14	18

Again, we eliminate variants 0 and 1, since $B_2 < b_2 = 8$ for them.

Solve the upper-level estimation problem

$$Z_1(B_1) + Z_2(B_2) \to \min$$

subject to the constraint

$$B_1 + B_2 \geq 30 .$$

The solution can be found in Table 20.

Table 20

5;18	9;32			
4;14	7;29			
4;12	6;27	8;33		
2	5;25	7;29	8;35	9;37
2 / 1	22;15	34;21	45;27	66;27

Consider Table 20 and choose a cell with the minimum first value among all cells whose second value is not smaller than $B = 30$. These are cells (8;35) and (8;33) with costs 8. According to the fundamental theorem of network programming, in our example costs 8 provide a lower estimate of the costs in the original problem. Define the corresponding optimal solutions by the backward method. Cell (8;35) corresponds to variant 2 in Table 19 and variant 4 in Table 18. Next, variant 2 in Table 19 answers the solution of the estimation problem for direction 2:

$x_2 = 0$, $x_3 = 0$, $x_4 = 1$.

Variant 5 in Table 18 answers to the solution of the first estimation problem:

$x_1 = 1$, $x_2 = 1$, $x_3 = 1$.

The obtained pair of solutions does not define an admissible solution.

Cell (8;33) corresponds to variant 3 in Table 19 and variant 3 in Table 18. Variant 3 in Table 19 answers to the solution

$x_2 = 1$, $x_3 = 0$, $x_4 = 1$

of the second estimation problem, whereas variant 3 in Table 18 corresponds to the solution

$x_1 = 1$, $x_2 = 0$, $x_3 = 1$

of the first estimation problem. Again, this pair of solutions does not define an admissible solution of the original problem (it represents a lower estimate only).

To proceed, we may either improve the derived estimates (using other costs splitting for multipurpose projects) or apply the branch-and-bound method with the derived estimates. Let us illustrate the branch-and-bound method. Choose direction 2 for branching. Decompose the solution set into two subsets: $x_2 = 1$(subset 1) and $x_2 = 0$ (subset 2).

Estimation on subset 1 ($x_2 = 1$).

As far as $x_2 = 1$, then

$B' = 30 - 10 = 20$, $b_1' = 10 - 6 = 4$, $b_2' = 8 - 4 = 4$.

Solve the estimation problem for direction 1:

$3x_1 + x_3 \to \min$

subject to the constraint

$12x_1 + 9x_3 \geq B_1'$,

where $4 \leq B_1' \leq 20$. The solution is described by Table 21.

Table 21

Variant	0	1	2	3
Z_1	0	1	3	4
B_1	0	9	12	21

Solve the estimation problem for direction 2:

$3x_3 + 3x_4 \to \min$

subject to the constraint

$6x_3 + 8x_4 \geq B_2$,

where $4 \leq B_2 \leq 20$. The solution is shown by Table 22.

Table 22

Variant	0	1	2
Z_1	0	3	6
B_1	0	8	14

Solve the upper-level estimation problem:

$Z_1(B_1) + Z_2(B_2) \to \min$

subject to the constraint

$B_1 + B_2 \geq 20$.

The solution is shown by Table 23.

Table 23

26;14	7;23	–	–
13;8	4;17	6;20	–
$Z_2;B_2$ \ $Z_1;B_1$	11;9	23;12	34;21

Actually, the solution answers to cell (6;20).

The first and second estimation problems have the solutions $x_1 = 1$, $x_3 = 0$, and $x_3 = 0$, $x_4 = 1$ respectively.

Note that the above pair of solutions defines an admissible, ergo optimal solution on the subset $x_2 = 1$ c with costs 8.

Estimation on subset 2 ($x_2 = 0$).

Solve the estimation problem for direction 1:

$3x_1 + x_3 \to \min$

subject to the constraint

$12x_1 + 9x_3 \geq B_1'$,

where $10 \leq B_1 \leq 30$. The solution is shown by Table 24.

Table 24

Variant	2	3
Z_1	3	4
B_1	12	21

Solve the estimation problem for direction 2:

$3x_3 + 3x_4 \to \min$

subject to the constraint

$6x_3 + 8x_4 \geq B_2$,

where $8 \leq B_2 \leq 30$. The solution is shown by Table 25.

Table 25

Variant	2	3
Z_2	3	6
B_2	8	14

Solve the upper-level estimation problem. The solution is given by Table 26.

Table 26

36;14	5;18	9;26	10;35
2 3;8	4;14	6;20	7;29
$Z_2;B_2$ \ $Z_1;B_1$	1 1;6	23;12	34;21

The solution answers to cell (10;35) with costs 10.

Choose subset 1 ($x_2 = 1$). The corresponding optimal solution is $x_1 = 1$, $x_2 = 1$, $x_3 = 0$, $x_4 = 1$ with costs 8. Fig. 3 shows the branching tree.

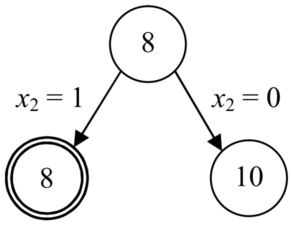

Fig. 3.

The second solution method of the problem consists in maximum increase of the lower estimate via optimal split of the costs c_2 and c_3 under the constraints

$s_{21} + s_{22} = c_2$,

$s_{31} + s_{32} = c_3$.

This is the so-called generalized dual problem (GDP).

According to [3], the GDP represents a convex programming problem. However, one should have in mind a couple of important aspects. First, numerical experiments have demonstrated that, generally, computational time required for lower estimate improvement is not compensated owing to smaller branching in the branch-and-boundary method. Second, in many cases the GDP possesses a non-integer solution; as is well-known, non-integer parameters make the knapsack problem NP-complex. Therefore, it is strongly recommended to obtain estimates under a given initial costs split of multipurpose projects.

We endeavor to improve the derived estimate. For $s_{21} = s_{22} = 1$, $s_{31} = 3$, there are two pairs of solutions to the estimation problems. The first pair of solutions has the form

$x_1 = 1, x_2 = 1, x_3 = 1,$

$x_2 = 0, x_3 = 0, x_4 = 1.$

And the second pair of solutions is defined by

$x_1 = 1, x_2 = 0, x_3 = 1,$

$x_2 = 1, x_3 = 0, x_4 = 1.$

Designate by δ_2 and δ_3 the variations of the estimates s_{22} and s_{32}, respectively. Note that the optimal solutions of the estimation problems remain same under small values of δ_2 and δ_3. To increase the lower estimate, we should increase the lower estimate for each pair of solutions.

The variations meet the inequalities $\delta_2 + \delta_3 > 0$ (for the first pair) and $\delta_2 - \delta_3 > 0$ (for the second pair). Choose $\delta_2 = 0$ and $\delta_3 > 0$. Interestingly, under $\delta_3 > 0$ we obtain a new pair of the optimal solutions to the estimation problems:

1) $x_1 = 1, x_2 = 1, x_3 = 0$,

2) $x_2 = 1, x_3 = 0, x_4 = 1$

associated with costs 8.

This pair defines an admissible, hence optimal solution of the original problem.

4. Joint financing mechanisms

A major problem in distributed project and program management lies in (financial) resource allocation among directions (subprograms) of a functionally distributed program or among separate departments (subprograms) of an administratively distributed program.

Consider a class of interests' coordination mechanisms for the Principal and agents. The matter concerns joint financing mechanisms of subprograms: a share of resources is provided by the Principal and the rest resources are contributed by agents. Here the subject of interests' coordination is the norm X defining the amount of Principal's resources allocated per agent's unit resources [4].

Our analysis begins with a simple analytical model. Suppose that the goal functions of agents take the form

$$f_i\left(x_i,\lambda\right) = 2\sqrt{r_i(1+\lambda)x_i} - x_i,\ i = \overline{1,m}, \quad (14)$$

where x_i is the amount of resources allocated to the subprogram by agent i. Under a given norm λ, each agent maximizes the goal function (14) with respect This problem has the solution

$$x_i = r_i\left(1+\lambda\right), i = \overline{1,m}. \quad (15)$$

The norm λ is defined by the limited resource condition of the Principal:

$$\lambda\left(1+\lambda\right) = \frac{R}{H}, \text{ where } H = \sum_i r_i.$$

Direct solution of this quadratic equation yields:

$$\lambda = \frac{1}{2}\left(\sqrt{1+4q} - 1\right), \text{ where } q = \frac{R}{H}.$$

For resource allocation, the Principal receives agents' estimates s_i of the efficiency levels r_i. Based on these data, the Principal evaluates

$$x_i = s_i\left(1+\lambda\right), \text{ where } \lambda = \frac{R}{S},\ S = \sum_i s_i.$$

Substitute x and λ into (15) to obtain

$$f_i = \left(1+\lambda\right)\left[2\sqrt{r_i s_i} - s_i\right]. \quad (16)$$

Under a large number of agents, the estimate provided by agent i has almost no influence on the norm λ. Let us accept the hypothesis of weak contagion (all agents neglect the above influence) and maximize the function (16) with respect to s_i. We naturally establish that $s_i = r_i,\ i = \overline{1,m}$. Thus, the joint financing mechanism enjoys strategy-proofness.

Now, switch to the discrete-time model. Assume that n_j projects exist for each subprogram. Each project yields the effect a_{ij} and incurs the costs c_{ij}, $i = \overline{1,n_j}$, $j = \overline{1,m}$.

Under the norm λ agent j invests $\frac{c_{ji}}{1+\lambda}$ in project i which gains the profit, $\pi_{ji} = a_{ji} - \frac{c_{ji}}{1+\lambda}$.

Obviously, if $\pi_{ji} > 0$, project i is included in the program. We believe that project i is also included in the program provided that $\pi_{ji} = 0$ (owing to agents' benevolence towards the Principal). Denote by $Q_j(\lambda)$ a set of projects such that $\pi_{ji} \geq 0$ under the norm λ. Find the maximum value of λ satisfying the inequality

$$\sum_{j=1}^{m} \sum_{i \in Q_J(\lambda)} c_{ji} \leq \left(1+\frac{1}{\lambda}\right) R. \quad (17)$$

Using such norm λ, the Principal participates in joint financing of all projects with nonnegative profits. Note that the Principal can choose a norm $\lambda > \lambda_0$. This leads to the problem of program design with the maximum total effect under a guaranteed effect of each agent. Actually, the problem has been studied above.

To solve inequality (17), for each project defines the norm

$$\lambda_{ji} = \frac{c_{ji}}{a_{ji}} - 1$$

(here $c_{ji} \geq a_{ji}$, otherwise, the project is beneficial to the agent without additional financing). Renumber all projects in the ascending order of λ_{ji}, i.e., $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_q$, where q means the number of projects.

Determine maximum number k such that.

$$\sum_{i=1}^{k} c_i \leq \left(1 + \frac{1}{\lambda_k}\right) R \,. \quad (18)$$

The obtained value λ_k provides a solution of inequality (17).

Remark. By assumption, for each agent $Q_j(\lambda_k) \neq \varnothing$ and, moreover, there exist projects with a guaranteed effect to the agent. If not, an agent should design projects with a sufficiently high effect.

Example 1. Take 2 subprograms and 2 agents, each having 4 projects. The corresponding parameters are combined in Table 27.

Table 27

i	1	2	3	4	5	6	7	8
a_i	100	50	80	60	40	30	70	20
c_i	110	60	104	84	60	48	119	36
λ_i	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8

Suppose that projects 1–4 (projects 5–8) are candidates for inclusion in subprogram 1 (subprogram 2, respectively). Set $R = 140$ and calculate

$\lambda = \lambda_1 = 0{,}1;\ c_1 < (1 + 10)R.$

$\lambda = \lambda_2 = 0{,}2;\ c_1 + c_2 < (91 + 5)R.$

$\lambda = \lambda_3 = 0{,}3;\ c_1 + c_2 + c_3 < (1 + 10/3)R.$

$\lambda = \lambda_4 = 0{,}4;\ c_1 + c_2 + c_3 + c_4 < (1 + 5/2)R.$

$\lambda = \lambda_5 = 0{,}5;\ 110 + 60 + 104 + 84 + 60 < 3R = 420.$

$\lambda = \lambda_6 = 0{,}6;\ 418 + 48 = 466 > 2{,}67 \times 140.$

Hence, $k = 5$ and the desired norm makes up $\lambda_5 = 0{,}5$.

In this case, subprogram 1 includes all projects 1–4, whereas subprogram 2 consists of project 5 only. However, if the guaranteed effects of the subprograms $d_1 = d_2 = 50$, then the plan coordination condition breaks for subprogram 2. Therefore, we choose $\lambda_k = \lambda_6 = 0{,}6$. As a result, subprogram 2 includes projects 5 and 6, with the total effect $a_5 + a_6 = 70 > 50$ and costs 80 of agent 2.

To design subprogram 1, solve the optimization problem

$100x_1 + 50x_2 + 80x_3 + 60x_4 \to \max$

subject to the constraint

$100x_1 + 60x_2 + 104x_3 + 84x_4 \leq 245^1/_3$

The optimal solution is

$x_1 = 1,\ x_2 = 0,\ x_3 = 1,\ x_4 = 0$

leading to effect 180.

Note that if we reduce the guaranteed effect of subprogram 2 to 40, then the norm λ goes down to 0.5. This allows increasing appreciably the total effect (from 330 to 250).

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ПРИНЦИП СКООРДИНИРОВАННОГО ПЛАНИРОВАНИЯ В УПРАВЛЕНИИ РАСПРЕДЕЛЕННЫМИ ПРОЕКТАМИ И ПРОГРАММАМИ

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Рассматриваются проблемы управления распределенными проектами и программами. Эти программы состоят из подпрограмм, распределенных функционально, в административном порядке или географически. Например, программа регионального развития включает в себя подпрограмму по экологической безопасности. В связи с этим основной проблемой управления распределенными программами является проблема координации интересов всех заинтересованных лиц. Мы предлагаем принцип скоординированного планирования для разработки планов реализации распределенных программ.

Ключевые слова: распределенные программы, экологическая безопасность, принцип скоординированного планирования.

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