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# THE ANALYSIS OF THESE PHYSIOLOGICAL SIGNALS ON THE PLANE OF COMPLEX FREQUENCIES WITH USE OF THE PRONI PROCEDURE

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The method of the spectral analysis on the plane of complex frequencies with use of the Proni procedure and also a graphic form of its representation is offered. In physiological studies and in assessing the functional state is very important to data analysis of physiological signals as oscillatory processes. In this widely accepted model of the oscillatory process is the representation of the process under study through the superposition (sum) of sinusoidal periodic processes, each of which is constant in time amplitude, frequency and initial phase. The calculated set of raw signal parameters harmonic sets the range of the investigated process. It should be noted that physiological signals in the General case, represent highly organized in time oscillations. Therefore, in the literature, physiological processes are mostly treated as random processes. In this case, as a measure of the distribution of the frequency of oscillatory activity of FS use a statistical evaluation of the power spectral density (PSD) of a signal, which reflects the dependence of the distribution of the average power (oscillatory activity) of the signal frequency

*Keywords: time row, Proni method, complex frequencies, spectral density. physiological signals, oscillatory processes, random processes, oscillatory activity.*

**Introduction**

In physiological researches and diagnostics of functional states the important place is taken by the analysis of these physiological signals (PS) as oscillatory processes. At the same time widespread model of oscillatory process is representation of the studied process through superposition (sum) of sinusoidal periodic processes, each of which is characterized by constants in time amplitude, frequency and an initial phase. A set of the parameters of harmonicas (sinusoidal periodic processes) calculated from an initial signal sets a range of the studied process (differently, distribution of amplitudes and phases of harmonicas depending on frequency). It should be noted that physiological signals generally represent the oscillatory processes which are difficult organized in time. Therefore in literature physiological processes generally are considered as casual processes. In this case as a distribution measure for the frequency of oscillatory activity of PS use statistical assessment of the spectral density of power (SPM) of a signal which reflects dependence of distribution on average of the power (oscillatory activity) of a signal from frequency.

**1. Framework for the analysis of SPM**

As a basic method for calculation of SPM of the studied signal the periodogrammny method of Welch, with the procedure of the fast transformation of Fourier (FTF) which is its cornerstone usually is used [1, 2]. For improvement of quality of the counted dependence of SPM of the studied process on frequency use also more difficult methods of calculation of SPM with use of procedures of parametrical modeling [1]. Initially concept SPM is entered as the characteristic of spectral structure of stationary casual process, that is process on average uniform in time (the invariance of dispersion, an average, etc.).

At the same time the dependence of SPM on frequency calculated for physiological non-stationary process on some final interval of time of process needs to be understood as an average spectral characteristic of this process on this interval of time. It is necessary to refer various transition processes arising in a human body to non-stationary processes (for example, transition processes of PS) [3, 4]. Temporary changes of the parameters causing not stationarity of process aren't reflected dependence of SPM.

Possible method of the analysis of a spectral characteristic of the studied non-stationary process is calculation of set of the dependences of SPM counted on short time intervals which are consistently displaced in time. However reduction of an interval of time for calculation of dependence of SPM leads to its washing out on frequency (to deterioration in resolution on frequency) and according to its quality [1]. For the analysis of a spectral characteristic of both stationary, and non-stationary FS the method of the spectral analysis on the plane of complex frequencies is offered. This method is realized in the computer program “Spectral Analysis of Physiological Signals” and underwent approbation at the solution of diagnostic and research tasks [4–7]. Also the possibilities of the analysis of SPM PS are considered by parametrical methods [6].

**2. Mathematical Model**

We will define a concept of complex frequency. Sinusoidal periodic harmonious process is expressed by the known formula:

$$X(t) = A\sin(2\pi f_0 t + \theta) = \mathrm{Im}\left[A\cos(2\pi f_0 t + \theta) + jA\sin(2\pi f_0 t + \theta)\right] =$$

$$= \mathrm{Im}\left[Ae^{i(2\pi f_0 t + \theta)}\right] = \mathrm{Im}\left[\dot{X}(t)\right], \tag{1}$$

where $A$ – amplitude; $f_0$ – cyclic frequency in hertz; $\theta$ – an initial phase in radians; $X(t)$ – instant value of process at the time of $t$; Im – an imaginary part of complex number; $j$ – imaginary unit; $\dot{X}$ – complex form of record process. Cyclic frequency of $f_0$ (number of cycles in unit of time) (1) is expressed by a formula [8]:

$$f_0 = \mathrm{Re}\left[(-j/2\pi)\left(d\dot{X}(t)/dt\right)\Big/\dot{X}(t)\right]. \tag{2}$$

Determine also circular frequency of $\omega_0$ connected with the cyclic frequency of $f_0$ $\omega_0 = 2\pi f_0$ ratio. At substitution of expression for $\dot{X}(t) = Ae^{j(2\pi f_0 t + \theta)}$ from (1) in (2) we will receive identity. We will consider non-stationary process of a look:

$$X(t) = Ae^{\alpha_0 t}\sin(2\pi f_0 t + \theta) = \mathrm{Im}\left[Ae^{\alpha_0 t}e^{j(2\pi f_0 t + \theta)}\right] = \mathrm{Im}\left[Ae^{j\left[(2\pi f_0 - j\alpha_0)t + \theta\right]}\right], \tag{3}$$

where $A(t) = Ae^{\alpha_0 t}$ – amplitude of sinusoidal process changing in time under the exponential law; $\alpha_0$ – attenuation coefficient (at $\alpha_0 > 0$ amplitude of $A(t)$ increases in time, $\alpha_0 < 0$ amplitude of $A(t)$ fades, at $\alpha_0 = 0$ process (3) turns into harmonious sinusoidal process with an invariable amplitude). Comparing expression (3) with (1) it is possible to determine complex circular frequency

$$\varpi = 2\pi f_0 - j\alpha_0 = \omega_0 - j\alpha_0. \tag{4}$$

It is possible to give strict definition of complex circular frequency of $\varpi$ according to expression (2), having omitted operation (Re) of capture of a material part:

$$\varpi = \left[-j\left(d\dot{X}(t)/dt\right)\Big/\dot{X}(t)\right]. \tag{5}$$

Substituting in (5) complex form of record of process (3)

$$\dot{X}(t) = Ae^{j(\omega_0 - j\alpha_0)t + \theta}$$

we will receive:

$$\varpi = -j\left(j\left[\omega_0 - j\alpha_0\right]\right) = \omega_0 - j\alpha_0, \tag{6}$$

at the same time

$$f_0 = (1/2\pi)\,\mathrm{Re}\,\varpi, \tag{7}$$

$$\alpha_0 = -\mathrm{Im}\,\varpi. \tag{8}$$

Thus, the complex circular frequency of $\varpi$ bears information as about the cyclic frequency of non-stationary process of $f_0$, and about coefficient of attenuation of $\alpha_0$, at change of amplitude of process under the exponential law. It should be noted that introduction of complex estimates increases their informational content [9]. For harmonious process of $\varpi = 2\pi f_0 = \omega_0$, owing to its stationarity the complex frequency of $\varpi$ coincides with the usual frequency of $\omega_0$. On the contrary, if process gets change of amplitude in time under the exponential law, then its frequency becomes complex size.

The reflecting non-stationary physiological process it is correct to present a signal through superposition of elementary sinusoidal processes, each of which is characterized by constants in time $f_0$ frequency, the initial phase $\theta$, initial amplitude And yes coefficient of attenuation of $\alpha_0$. In this case the non-stationary physiological signal decays on sinusoidal components with amplitudes changing in time under the exponential law and is presented in the range form on the plane of complex frequencies. At the same time the range on the plane of complex frequencies is generalization of a usual range and can serve for the analysis of the non-stationary processes reflecting including transitional states. As the basic procedure for calculation of such range the Proni procedure is used [1]. It is noted that the Proni method isn't among methods of spectral estimation [1]. However, definition of a concept of complex frequency of $\varpi$ (5) and its physical sense allow the Proni method to carry to a method of the spectral analysis on the plane of complex frequencies.

In the spectral analysis of a physiological signal on the plane of complex frequencies there is an approximation of the sequence from counting of process $x(1)$, $x(n)$ of a linear combination of cosinusoids, fading, increasing or invariable on amplitude:

$$X[n] = \sum_{K=1}^{L/2} 2A_k \exp\left[\alpha_k (n-1)T\right]\cos\left[2\pi f_k (n-1)T + \theta_k\right], \qquad (9)$$

where $1 < n < N$; $L$ – order of the approximating cosinusoidal model; $T$ – an interval of counting in seconds; $A_k$ and $\alpha_k$ – amplitude and coefficient of attenuation (in $s^{-1}$) $k$ of a cosinusoid; $f_k$ and $\theta_k$ – frequency (in Hz) and an initial phase (in rad) cosinusoid $k$. Parameters for plotting the spectrum on the complex frequency plane of the analyzing process are: $f_k$, $\alpha_k$, $R_k$ – the power of the $k$-th varying amplitude cosine wave, the index k varies from 1 to $L/2$. In Figs. 1 and 2 different variants of spectral analysis in the complex frequency plane of the transition process of the heart rhythm of the person resulting research are considered.

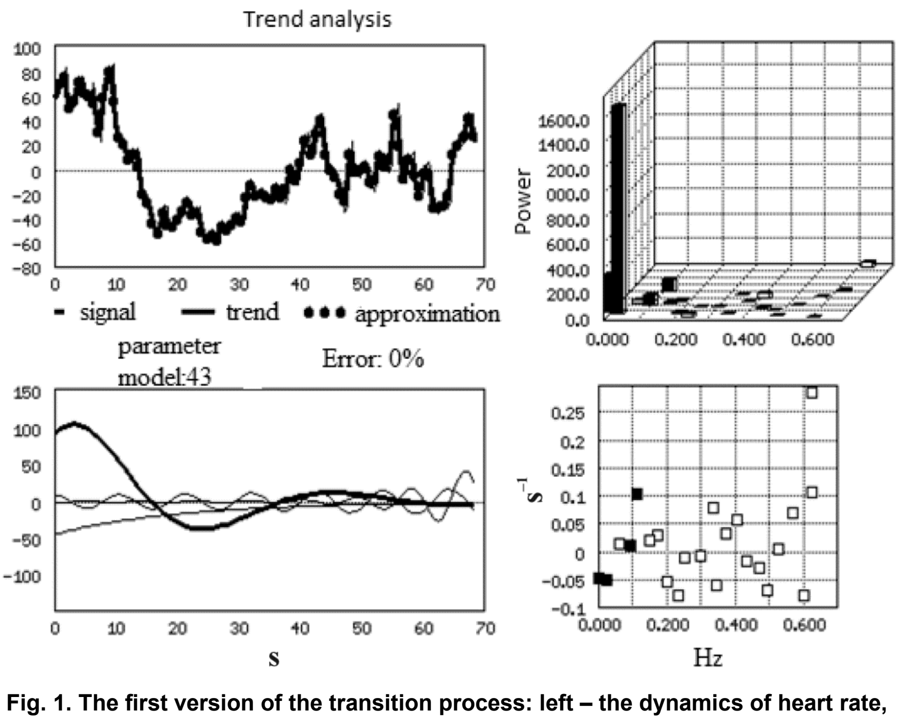
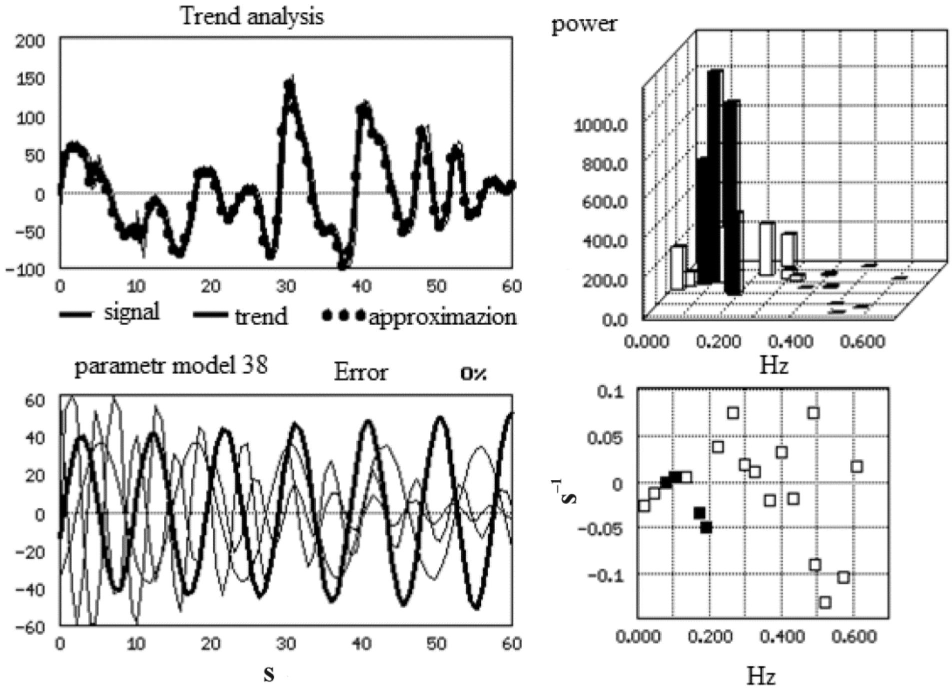



**Fig. 1. The first version of the transition process: left – the dynamics of heart rate, right – heart rate spectrum of the transition process in the complex frequency plane**




a) b)

**Fig. 2. Second option of transition process: a) dynamics of a warm rhythm; b) a range of a warm rhythm of transition process on the plane of complex frequencies**

In conclusion, we can note the following: 1) the range on the plane of complex frequencies as a frequency image of temporary dynamics of warm rhythm can be a basis at creation of system of classes of transition processes of a warm rhythm; 2) the generalized of complex frequencies parameters allow to estimate quantitatively reactivity of various mechanisms of regulation of warm rhythm in response to functional reflex types [10].

**Conclusions**

1. The range on the plane of complex frequencies as a frequency image of temporary dynamics of warm rhythm can be a basis at creation of system of classes of transition processes of a warm rhythm.

2. The generalized of complex frequencies parameters allow to estimate quantitatively reactivity of various mechanisms of regulation of warm rhythm in response to functional reflex types [10].

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# АНАЛИЗ ДАННЫХ ФИЗИОЛОГИЧЕСКИХ СИГНАЛОВ НА ПЛОСКОСТИ КОМПЛЕКСНЫХ ЧАСТОТ С ИСПОЛЬЗОВАНИЕМ ПРОЦЕДУРЫ ПРОНИ

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Предложен метод спектрального анализа на плоскости комплексных частот с использованием процедуры Прони, а также графическая форма его представления. В физиологических исследованиях и диагностике функциональных состояний важное место занимает анализ данных физиологических сигналов как колебательных процессов. При этом широко распространенной моделью колебательного процесса является представление исследуемого процесса через суперпозицию (сумму) синусоидальных периодических процессов, каждый из которых характеризуется постоянными во времени амплитудой, частотой и начальной фазой. Набор рассчитанных из исходного сигнала параметров гармоник задает спектр исследуемого процесса. Необходимо отметить, что физиологические сигналы в общем случае представляют собой сложно организованные во времени колебательные процессы. Поэтому в литературе физиологические процессы в основном рассматриваются как случайные процессы. В этом случае в качестве меры распределения по частоте колебательной активности физиологического сигнала используют статистическую оценку спектральной плотности мощности сигнала, которая отражает зависимость распределения в среднем мощности (колебательной активности) сигнала от частоты.

*Ключевые слова: временной ряд, метод Прони, комплексные частоты, спектральная плотность, физиологические сигналы, колебательные процессы, случайные процессы, колебательная активность.*

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