# Краткие сообщения

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### **AVERAGING AND MIXING**

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The article is devoted to the explanation of the role of averaging and mixing in applied problems in technology and economics. The article provides the examples of applied problems solved at the Mathematical Programming Department of the N.N. Krasovskii Institute of Mathematics and Mechanics and at the department of Econometrics and Statistics of the Graduate School of Economics and Management of the Ural Federal University named after the First President of Russia B.N. Eltsin.

These are the problems in geology, mining and metallurgy, placement and development of enterprises which were traditionally solved in the Urals. Besides, the problems in planning and operational management at ore-dressing and processing enterprises were solved. A new wave of operational management at enterprises started at the beginning of 2000's should be noted. This operational management applied the committee method.

Being mathematicians, we specified certain inaccuracies in publications made before 1990 on this subject. It is noteworthy that the principal performers of this research – the research workers of the Mathematical Programming Department – were awarded with the medals of Exhibitions of Economic Achievements of USSR.

The set of research done by N. Bourbaki's seminar participants was devoted to averaging and mixing. In general, the research performed at the Mathematical Programming Department and in N. Bourbaki's seminar provided a better understanding of material averaging and activity combination processes.

Modern applied research requires a more detailed solution substantiation in terms of pragmatics that was also noted in the article.

When solving these problems a method of committee construction was applied. This is also a fundamentally important approach to solving incompletely formalized problems of optimal decisions.

Keywords: charge mixture properties, composites, mixtures, committees, diagnostics, stability, selection, interpretation, statistics, balance.

#### Introduction

Over 50 years ago together with I.I. Eremin we started to research non-stationary optimization processes and nonintrinsic problems of mathematical programming that we had introduced [1]. Recently our research has been considered by philosophers. T.H. Kerimov writes in his book entitled "Unsolvabilities" ("Nerazreshimosty") [2]: "Unsolvable structures constituting diverse discourses are being analyzed".

As far back as 1970's and 1980's together with V.M. Kislyak we used to study models of mining and smelting industry typical for the Urals within the scope of research of the Mathematical Programming Department of the Institute of Mathematics and Mechanics [3–5]. We discussed the topic at the department conference and criticized A.N. Ramm's rough model of charge mixture preparation where charge mixture properties were produced by simple additive mixing of charge mixture components properties [6–8]. Therefore the article by H. Attouch, Homogenization – Sem. Bourbaki – 1988 drew my attention [9]. There H. Attouch relates about a more precise composite material modeling as before on the basis of mathematical physics methods. I got interested because I was researching mixture problems including mixed strategies in the games theory, in committee theory and also because the topic

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of H. Attouch's article was unusual for N. Bourbaki. Problems of mixing certain objects emerge quite often. They are not easy ones because as a rule there appear non-additivity.

As it turns out, besides the famous multivolume edition "Elements of mathematics" N. Bourbaki also publishes the seminar materials written in a "simple" language. H. Attouch in 1998 included the article "Averaging" to the seminar proceedings [10]. He marks the progress in composite materials research. The article establishes the principle of averaging for the partial differential equations. H. Attouch underlines that in tasks in physics most parameters are discontinuous and that constitutes a problem.

#### **Idea of committees**

In case of group or collective decision (like in committee methods) or in case of component mixing in charge mixtures additivity means that the productivity of collective decision constitutes the sum of reactions of every group member (every mixture component) [11].

Since we will also apply linguistics, it should be noted that E. Sapir and B.L. Whorf state in their hypothesis of "Linguistic relativity" that the language structure influences the world perception and outlook of its speakers.

When working with charge mixtures in metallurgy additivity is sometimes presupposed and therefore calculations of mathematical model are incorrect. In general, non-additivity occurs in chemical reactions [4, 12].

Examples: mixing of technologies, substances, elements of linear space and problems in medicine and biology, mixed strategies in game theory. Also biogeocenosis properties and even results of diet problems. Convex sets consist of mixtures of a certain set of elements.

#### **Specifics**

I intend to consider mixing processes from a certain general point as well as from that of specific applied problems.

The methods of mathematical programming, images recognition and factor analysis are used [13]. Factor naming procedure is given.

In the articles on components combination and mixing 'mixtures' are sometimes assumed as the property combinations. Needless to say that this assumption was not correct though for a certain wide overview of the region economy it was supposed as preliminary acceptable. It was based on A.N. Ramm model. H. Attouch suggested a more precise model in the article mentioned above.

One of the methods of mixture property recognition is neural network training on the basis of precedents. Mixture structures can be different: layer, fiber, porous even unsystematic at all.

Recognition of their properties is made on the basis of publications on mathematical physics methods and image recognition. Recognition enables to generate hypothetical rules of new mixture diagnostics based on training material. Practical methods of decision making are analyzed with the involvement of specialists.

Examples of mixture problems: diet choice, charge mixture properties, technology mix, decision making by groups of specialists.

The topic 'mixtures' includes mining technologies, ore geological properties, mixed strategies, in enrichment: chemical and mineralogical composition.

Collective decisions theory and mathematical statistics methods are applied.

More examples of applied problems: technical and economic analysis in solving mining operation modeling problems, production models of ferrous and non-ferrous metals.

A more general example was given by V.I. Katkovnik in his book "Linear estimates and stochastic problem of optimization". He suggested a method of parametric averaging operators. This method is applied when researching the systems the status of which are vectors x of linear space specified by neural network parameters and structure. In this case network operation is determined by vector [x, y], y – output: y = y(x).

## **Committee concept**

In its essence the idea of committee method is that of percentage of some special H subselection in G selection. This fact justifies the application of committee decisions in statistical problems. Besides, mathematical medicine involves micromedical models and macromedicine is a medical statistics.

# Краткие сообщения

### **Averaging stability**

An international conference on image recognition took place in 1980's in the city of Brody. My report on ambiguous interpretation of n-dimensional polyhedron images was highly appreciated by academician Y.I. Zhuravlev. A.B. Glaz also spoke there about formation of decision rules minimizing the average risk, the report concerned collectives of recognition decision rules. He was asked about the reference on Vl.D. Mazurov's research but he was not acquainted with it.

Though A.B. Glaz noted as well about averaging stability that is a typical situation in mathematical statistics. V.I. Arnold writes: 'The basics of the mixing theory in dynamic systems are stated in the research by Lagrange and Laplace'. Y.V. Chaikovsky in his article on statistical world outlook formed in 1920's also talks about averaging stability. E. Romanovsky published an article on this topic entitled 'Statistical world outlook' in 1922.

#### Statistical world outlook

The history of the world outlook formation is interesting. In 1920's the concept of "statistical world outlook" emerged. Statistics consists of average and balance. In the early history of statistics it was viewed as accounting, description and calculation. Further on, as statistical hypothesis verification through statistical tendency and correlation. Balance principle occurred in accounting, meanwhile in accounting the balance is aimed at that between receipts and expenditure, in case there is none, since the very presence of receipts and expenditure disturbs the equality of receipts to expenditure, a fictitious balance is introduced. Equilibrium concept is generalized in nonequilibrium dynamics.

Consider a system

$$X \in D_j \ (j=1,...,m). \tag{1}$$

In particular it can be a system

$$f_j(x) > 0 \ (j=1,...,m).$$
 (2)

This article has the following committee constructions:

MCS – maximal (in sense of including) consistent subsystem;

MIS – minimum (in sense of including) inconsistent subsystem.

Committee for system (1) is a finite sequence C where every j-condition is satisfied with most elements of C.

The well-known properties of committee constructions should be reminded of.

Theorem 1.

If a committee exists, there exists a committee consisting of decisions of MCS.

Theorem 2.

If each k-sets of system (1) intersects and k/m > p, then there exists p is a committee when p = k/m.

### **Conclusions**

- 1. Ramm's model for charge mixture calculation does not provide exact mixing results.
- 2. The model based on neural network and recognition is given in the article.
- 3. The suggested model is applied not only to the problem of charge mixture but also to a wide range of problems on averaging and mixing.
  - 4. Factor nomination should apply methods of mathematical linguistics.

### References

- 1. Eremin I.I., Mazurov Vl.D. *Nestatsionarnye protsessy matematicheskogo programmirovaniya* [Nonstationary Processes of Mathematical Programming]. Moscow, Nauka Publ., 1979. 291 p.
- 2. Kerimov T.K. *Nerazreshimosti*. [Unsolvabilities]. Moscow, Akademicheskiy proekt Publ., 2007. 218 p.
- 3. Khaykin V.P. *Korrelyatsiya i statisticheskoe modelirovanie v ekonomicheskikh raschetakh* [Correlation and Statistical Model Operation in Economic Calculations]. Moscow, Economica Publ., 1964. 213 p.
- 4. Benuni A.KH. *Vyyavlenie i ispol'zovanie proizvodstvennykh rezervov tsvetnoy metallurgii* [Identification and Use of Production Reserves of Nonferrous Metallurgy]. Moscow, Metallurgy Publ., 1962. 230 p.

- 5. Nemirovskiy A.S., Yudin D.B. *Slozhnost' zadachi*... [Complexity of a Task ...]. Moscow, Nauka Publ., 1979. 384 p.
- 6. Esipov N., Tyagunov L.I. [Definition of Rational Extent of Enrichment of Coals. Solution of Applied Tasks: Forecasting of Quality of Coke by Method of Committees]. *Method of Committees in Recognition of Images: Coll. of Works*, Sverdlovsk, 1974, iss. 6, pp. 131–146. (in Russ.)
- 7. Borel E. Sur les principles de la theorie cinetique des gaz. *Annales scientifiques de l'Ecole normale superieure*, 1906, 23, pp. 9–32.
- 8. Ramm A.N. *Opredelenie tekhnicheskikh pokazateley domennoy plavki. Metodicheskoe rukovodstvo* [Definition of Technical Indexes of Domain Melting. Methodical Manual]. Leningrad, LPI Publ., 1971. 215 p.
- 9. Attouch H. Homogénéisation. *Séminaire Bourbaki*: volume 1987/88, exposés 686–699, *Astérisque*, no. 161–162 (1987–1988), Talk no. 686, pp. 7–30. Available at: http://www.numdam.org/article/SB 1987–1988 30 7 0.pdf.
- 10. Smirnov A.I. *Nekotorye modeli biologicheskoy dinamiki: kand. dis.* [Some Models of Biological Dynamics. Cand. Sci. Diss.]. Sverdlovsk, 1985. 112 p.
- 11. Mazurov Vl.D. *Metod komitetov v zadachakh optimizatsii i klassifikatsii* [Method of Committees in Problems of Optimization and Classification]. Moscow, Nauka Publ., 1990. 248 p.
- 12. Benuni A.Kh., Gurfel' B.L. *Matematicheskie metody v planirovanii i upravlenii tsvetnoy metallurgii* [Mathematical Methods in Scheduling and Management of Nonferrous Metallurgy]. Moscow, Metallurgy Publ., 1974. 191 p.
- 13. Katkovnik V.Ya. *Lineynye otsenki i stokhasticheskie zadachi optimizatsii* [The Linear Estimators and Stochastic Problems of Optimization]. Moscow, Nauka Publ., 1976. 488 p.

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# УСРЕДНЕНИЕ И СМЕШИВАНИЕ

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Статья посвящена объяснению роли усреднения и смешивания в прикладных задачах техники и экономики. Приводятся примеры прикладных задач, решённых в отделе математического программирования Института математики и механики имени Н.Н. Красовского Уральского отделения Российской академии наук, а также на кафедре эконометрики и статистики Высшей школы экономики и менеджмента Уральского федерального университета имени первого Президента Российской Федерации Б.Н. Ельцина.

Это задачи геологии, горного дела и металлургии, размещения и развития соответствующих предприятий, традиционно решающиеся на Урале. Кроме того, решались задачи планирования и управления работами горно-обогатительных комбинатов. Надо отметить новую волну работ управления предприятиями, начавшуюся в начале 2000 годов. Это работы, использующие метод комитетов.

При этом мы как математики указали на некоторые неточности в публикациях до 1990 года по этой тематике. Заметим, что основные исполнители этих работ — сотрудники отдела математического программирования — получили медали Выставки достижений народного хозяйства СССР.

Тематике усреднения и смешивания был посвящён комплекс работ участников семинара Бурбаки. В целом работы отдела математического программирования и семинара Бурбаки послужили лучшему пониманию процессов смешивания материалов и комбинирования активностей.

## Краткие сообщения

Современные прикладные работы требуют более детального обоснования решений с точки зрения прагматики. Этому тоже уделено внимание.

В процессах решения этих задач был применён метод комитетных конструкций. Это тоже принципиально важный подход к решению неполностью формализованных задач оптимального выбора решений.

Ключевые слова: свойства шихты, композиты, смеси, комитеты, диагностика, устойчивость, выборка, интерпретация, статистика, баланс.

### Литература

- 1. Ерёмин, И.И. Нестационарные процессы математического программирования / И.И. Ерёмин, Вл.Д. Мазуров. М.: Наука, 1979. 291 с.
  - 2. Керимов, Т.К. Неразрешимости / Т.К. Керимов. М.: Академический проект, 2007. 218 с.
- 3. Хайкин, В.П. Корреляция и статистическое моделирование в экономических расчётах / В.П. Хайкин. M.: Экономика, 1964. 213 с.
- 4. Бенуни, A.X. Выявление и использование внутрипроизводственных резервов цветной металлургии / A.X. Бенуни. M.: Металлургия, 1962. 230 с.
- 5. Немировский, А.С. Сложность задачи... / А.С. Немировский, Д.Б. Юдин. М.: Наука, 1979. 384 с.
- 6. Есипов, Н. Определение рациональной степени обогащения углей. Решение прикладных задач: прогнозирования качества кокса методом комитетов / Н. Есипов, Л.И. Тягунов // Метод комитетов в распознавании образов: сб. ст. – Свердловск, 1974 – Вып. 6. – С. 131–146.
- 7. Borel, E. Sur les principles de la theorie cinetique des gaz / E. Borel // Annales scientifiques de l'Ecole normale superieure. -1906. -23. -P. 9-32.
- 8. Рамм, А.Н. Определение технических показателей доменной плавки / А.Н. Рамм. Л.: Политехнический институт, 1971. 215 c.
- 9. Attouch, H. Homogénéisation / H. Attouch // Séminaire Bourbaki: volume 1987/88, exposés 686–699; Astérisque, no. 161–162 (1987–1988), Talk no. 686, p. 7–30. http://www.numdam.org/article/SB 1987–1988 30 7 0.pdf.
- 10. Смирнов, А.И. Некоторые модели биологической динамики: канд. дис. / А.И. Смирнов. Свердловск: Ур $\Gamma$ У, 1985. 112 с.
- 11. Мазуров, Вл.Д. Метод комитетов в задачах оптимизации и классификации / Вл.Д. Мазуров. М.: Наука, 1990. 248 с.
- 12. Бенуни, А.Х. Математические методы в планировании и управлении цветной металлургии / А.Х. Бенуни, Б.Л. Гурфель. М.: Металлургия, 1974. 191 с.
- 13. Катковник, В.Я. Линейные оценки и стохастические задачи оптимизации / В.Я. Катковник. М.: Наука, 1976. 488 с.

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