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EXISTENTIAL ISSUES OF COMMITTEE CONSTRUCTIONS. PART II

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The question of the existence of a committee of a system of linear inequalities under additional conditions is considered. The most part of the article is devoted to the results of the research conducted by Vl.D. Mazurov and M.Y. Khachai on the committees of systems of linear inequalities. The given article represents the continuation of the results. The question of the proofs of the results in infinite-dimensional case is answered. This is the most difficult part of the problem.

The committee of a system of algebraic inequalities is an ordered set of decision rules on the basis of which the final procedure of decision making is formed.

The problem of committee construction and their application in economics and technics is topical since their initial formulation often contains controversies and non-formalized parts. Therein the system of homogeneous linear inequalities with an infinite set of indices is considered. Solution set can be empty as well. The conditions are proved under which there exist a committee of the system. As it follows from the theorem when the number of limit points in the left parts of inequalities is finite then the problem is reduced to that of construction of mutually independent committees. The example is given.

At present factor analysis with the similar features is becoming increasingly important, the given mathematical apparatus can be applied to them as well.

Further on, these methods are applied in psychology including a depth one the research of which was initiated by Carl Jung.

Keywords: recognition, inequality system, set separation.

Introduction

The subject matter of the article is topical what can be proved by the recently published works by F.P. Chernavin on the application of committee constructions in business and those by D.V. Gilev on mathematical medicine. The crux of the problem is that it is necessary to apply more complex decision rules. In particular, the exact theorems of committee methods justification are proved. These methods are valuable due to the opportunity of solving more complex problems of decision making. The proofs by M.Yu. Khachay are cited and included in the text of the article.

1. Sufficient conditions of committee solvability

This part refers to a system of homogeneous linear inequalities

$$(c_\alpha, x) > 0 \ \ (\alpha \in M), \tag{1.1}$$

where $x, c_\alpha \in R^n, M$ – an infinite set of indices. Before formulating and proving the theorem, stating the sufficient conditions of existence of (1.1) system committee, let us prove two preliminary lemmas of a relatively general nature.

Lemma 1.1. Suppose a finite system $(c_j, x) > 0$ $(j \in N_m)$ satisfies the condition $\forall \lambda \in R, \forall i, j \in$ $\in N_m$ $c_j + \lambda c_i \neq 0$.

Therefore for any $K = (k_1, k_2, \dots, k_m)$ sequence consisting of m natural numbers of the same parity, a committee $Q = \{x^1, \dots, x^q\}$ will be found such that $\forall j \in N_m \left|\{i: (c_j, x^i) > 0\}\right| - \left|\{i: (c_j, x^i) > 0\}\right| -$ $-\left|\{i: (c_j, x^i) < 0\}\right| = k_j$, that is, in other words, there will be found a committee voting for j-inequality with k_j vote edge.

Proof. 1st case. All numbers $k_1, \ldots, k_m$ are even. Let us apply to our system, which is committee solvable according to the statement, the algorithm of committee construction based on the proof of existence theorem [4, 5] according to which for any c_j vector the x^j point is found such that $(c_j, x^j) = 0$, $(c_i x^j) \neq 0$ $(i \neq j)$. As a result a set $Q = \{x^1 + \varepsilon c_1, -x^1 + \varepsilon c_1, \ldots, x^m + \varepsilon c_m, -x^m + \varepsilon c_m\}$ at sufficiently small ε is a system committee, vectors $x^j + \varepsilon c_j, -x^j + \varepsilon c_j$ satisfying *j*-inequality, and exactly one vector from each pair $x^i + \varepsilon c_j, -x^i + \varepsilon c_j$ $(i \neq j)$ satisfying it.

Let us consider a new set Q' which is different from Q in that *j*-pair of elements from Q is reproduced $k_j/2$ times. This Q' set is the required committee because it contains $k_1 + \cdots + k_m$ members out of which $2*(k_j/2) + k_1/2 + \cdots + k_{j-1}/2 + k_{j+1}/2 + \cdots + k_m/2$ vectors satisfy *j*-inequality, therefore $\left|\{i: (c_j, x^i) > 0\}\right| - \left|\{i: (c_j, x^i) \leq 0\}\right| = k_j$, as was to be proved.

2nd case. All numbers $k_1, \ldots, k_m$ are odd. Let us consider a Q committee, obtained by means of the method of projection on the plane [5, 6]. Then the committee will have the following property:

Q has an odd number of members (this number corresponds to the number of projection of maximum (in sense of including) inconsistent subsystem, $k + 1$ vectors out of $2k + 1$ vectors of Q committee satisfying *j*-inequality. Let us consider the sequence of even numbers $k_1 - 1, \ldots, k_m - 1$. According to the 1st case, there will be found a set $Q' = \{x^1, \ldots, x^q\}$ such that $\forall j \in N_m \left|\{i: (c_j, x^i) > 0\}\right| - -\left|\{i: (c_j, x^i) \leq 0\}\right| = k_j - 1$.

Combining sets Q and Q' we will obtain the required system committee. The lemma is proved.

Comment 1.1. It is easily seen that the requirement of numbers $k_1, k_2, \ldots, k_m$ positivity is not essential. It is sufficient to consider arbitrary integral numbers of the same parity, in this case the set obtained will not represent a committee.

Lemma 1.2. Suppose a set of vectors $C \subset R^n$ satisfies the conditions

1) $\forall \lambda \in R, \forall c', c'' \in C \quad c' + \lambda c'' \neq 0$.
2) The number of limit points of set $\{c/\|c\| : c \in C\}$ is finite. Then for every vector c^* such that $c^*/\|c^*\|$ is an isolated point of set $\{\pm c/\|c\| : c \in C\}$ there will be found two vectors h_1, h_2 such that

1) $(h_1, c^*) > 0, ((h_2, c^*) > 0$;
2) $\forall d \in C\ (d \neq c^*)\ ((h_1, d) > 0, (h_2, d) \leq 0\ or\ (h_1, d) \leq 0, (h_2, d) > 0$.

Proof. Without limiting the generalities, we will consider that the norms of all vectors from *C* are equal to 1. Let us consider an arbitrary isolated point $c^* \in C$. Suppose $c_1, c_2, \ldots, c_m$ are limit points of C set. It is evident that $\|c_1\| = \|c_2\| = \cdots = \|c_m\| = 1$ and $\forall i c^* \neq \pm c_i$. Then it is evident that $\exists h \in R^n: (h, c^*) = 0, (h, c_1) \neq 0\ i \in \overline{1, m}$.

Let us denote $\lambda = inf|(h, c)|$ where $c \in C \backslash \{c: (h, c) = 0\}$. Let us demonstrate that $\lambda > 0$ by contradiction. Suppose $\lambda = 0$. Then there can be found such a sequence c_n such that $(h, c_n) \to 0$ with $n \to \infty$ and $\forall n (h, c_n) \neq 0$. Since $\{c_n\} \subseteq C$ and C is limited (norms of all vectors are equal to 1), then $c_n \to c_0$ with $n \to \infty$. Therefore, c_0 is a limit point of C set and at the same time due to continuity of scalar product $(h_n, c_0) = 0$. It contradicts the choice of vector *h*.

Let us note that the number of points from *C* belonging to hyperplane $\{x: (h, x) = 0\}$ is not more than finite because otherwise there can be found a limit point of *C* set belonging to hyperplane and that is not possible. Suppose $d_1, d_2, \ldots, d_r$ are the points from *C* such that $(h, d_i) = 0$. Since c^* is not collinear with d_i then [3] $\exists p \in R^n: (p, c^*) = 0, (p, d_i) \neq 0\ \ i \in \overline{1, r}$.

Let us consider vector $h + \varepsilon_1 p$, where $0 < \varepsilon_1 < \lambda/\|p\|$. Then firstly, $(h + \varepsilon_1 p, c^*) = (h, c^*) + +\varepsilon_1(p, c^*) = 0$. Secondly, $\forall c \in \overline{C} \backslash \{c^*\} \quad h + \varepsilon_1 p \neq 0$.

Indeed, for points belonging to hyperplane $\{x: (h, x) = 0\}$ we have $(h + \varepsilon_1 p, d_i) = (h, d_i) + +\varepsilon_1(p, d_i) = \varepsilon_1(p, d_i) \neq 0$. If $c \in \overline{C} \backslash \{c: (h, c) = 0\}$ then $|(h + \varepsilon_1 p, c)| = |(h, c) + \varepsilon_1(p, c)| \geq \geq |(h, c)| - \varepsilon_1|(p, c)| \geq \lambda - \varepsilon_1\|p\| > 0$. Let us denote $\lambda_2 = min|(h + \varepsilon_1 p, c)|, c \in \overline{C} \backslash \{c^*\}$. Since $\overline{C} \backslash \{c^*\}$ is compact (c^* is an isolated point), then $\lambda_2 > 0$. Let $h_1 = h + \varepsilon_1 p + \varepsilon_2 c^*, h_2 = -h - \varepsilon_1 p + \varepsilon_2 c^*$, where $0 < \varepsilon_2 < \lambda_2$. Let us demonstrate that h_1, h_2 are the required ones. Actually $(h_1, c^*) = = (h + \varepsilon_1 p, c^*) + \varepsilon_2(c^*, c^*) = \varepsilon_2 > 0$. By analogy $(h_2, c^*) > 0$. Let us consider now an arbitrary point $c \in C \backslash \{c^*\}$. Since $|(h + \varepsilon_1 p, c)| \geq \lambda_2 > \varepsilon_2 \geq \varepsilon_2|(c^*, c)|$, then numbers (h_1, c) and (h_2, c) have a different sign. Therefore, vectors h_1, h_2 are the required ones. The lemma is proved.

Theorem 1.1. Suppose for a system $(c_\alpha, x) > 0$ $(\alpha \in M)$ the following conditions are met:

1) Among vectors c_α there exist no null or opposite ones.
2) The number of limit points of set $\{c_\alpha/\|c_\alpha\|\}$ is not more than finite.
3) If $c, -c \in \overline{\{c_\alpha/\|c_\alpha\|\}}$ then with sufficiently small $\varepsilon > 0$ there exists a committee $Q = \{x^1, \dots, x^q\}$ of subsystem $(c_\alpha, x) > 0, \alpha \in M(c, -c, \varepsilon)$ such that $\forall d \neq c: d, -d \in \overline{\{c_\alpha/\|c_\alpha\|\}}$ $(x^j, d) \neq 0$.

Then there exists a system committee $(c_\alpha, x) > 0$ $(\alpha \in M)$.

Proof. Suppose $d_1, \dots, d_r, c_1, -c_1, \dots, c_m, -c_m$ are all limit points of set $\{c_\alpha/\|c_\alpha\|\}$. According to condition 3) for every pair $c_i, -c_i$there will be found a set of points Q_i which represent at certain $\varepsilon_i > 0$ a subsystem committee $(c_\alpha, x) > 0, \alpha \in M(c_i, -c_i, \varepsilon_i)$. We will suppose that ε is the same for all committees (it is suffice to let $\varepsilon_0 = min\varepsilon_1$). Let us calculate the impact of the combined set $K_j = Q_1 \cup Q_{j-1} \cup Q_{j+1} \cup \dots \cup Q_m$ on *j*-point c_j notably: let us assign number k_j defined in the following way to every vector c_j:

$k_j = \left|\{x \in K_j: (c_j, x) > 0\}\right| - \left|\{x \in K_j: \left(c_j, x\right) \leq 0\}\right|$ $j \in \overline{1, m}$.

We can suppose that the number of members in any committee Q_j is odd, then it is easily seen that all numbers $k_1, k_2, \dots, k_m$ have the same parity (dependent on *m*). Since among vectors $c_1, c_2, \dots, c_m$ there are no null or collinear ones, then we have a right to apply lemma 1.1 according to which there will be found a set Q^* such that:

$\left|\{y \in Q^*: (c_j, y) > 0\}\right| - \left|\{y \in Q^*: \left(c_j, y\right) \leq 0\}\right| = -k_j \quad j \in \overline{1, m}$.

From the theorem statement and lemma 1 conclusion it follows that

$\forall y \in K_j \cup Q^* \quad \left(c_j, y\right) \neq 0 \; j \in \overline{1, m}$.

Since the set of vectors $K_j \cup Q^*$ is finite, then for the sufficiently small $\varepsilon_1 > 0$ the following statement is correct $\forall x \in O_{\varepsilon 1}\left(c_j\right), \forall y \in K_j \cup Q^*$ $sgn(x, y) = sgn\left(c_j, y\right)$ $j \in \overline{1, m}$.

Suppose that $\varepsilon_1 < \varepsilon_0$. Let us consider a set $Q = Q^* \cup Q_1 \cup \dots \cup Q_m$. Let us demonstrate that Q is a committee of a subsystem $(c_\alpha, x) > 0, \alpha \in M(c_1, -c_1, \varepsilon_1) \cup \dots \cup M(c_m, -c_m, \varepsilon_1)$. Indeed, suppose c_α is such that $\alpha \in M\left(c_j, -c_j, \varepsilon_1\right)$ for some $j \in \overline{1, m}$. Then if $\frac{c_\alpha}{\|c_\alpha\|} \in O_{\varepsilon 1}\left(c_j\right)$, then

$$
\begin{aligned}
&|\{y \in Q: (c_\alpha, y) > 0\}| - |\{y \in Q: (c_\alpha, y) \leq 0\}| = |\{y \in Q^*: (c_\alpha, y) > 0\}| - \\
&-|\{y \in Q^*: (c_\alpha, y) \leq 0\}| + \left|\{y \in K_j: (c_\alpha, y) > 0\}\right| - \left|\{y \in K_j: (c_\alpha, y) \leq 0\}\right| + \\
&+\left|\{y \in Q_j: (c_\alpha, y) > 0\}\right| - \left|\{y \in Q_j: (c_\alpha, y) \leq 0\}\right| = \left|\{y \in Q^*: \left(c_j, y\right) > 0\}\right| - \\
&-\left|\{y \in Q^*: \left(c_j, y\right) \leq 0\}\right| + \left|\{y \in K_j: \left(c_j, y\right) > 0\}\right| - \left|\{y \in K_j: \left(c_j, y\right) \leq 0\}\right| + \\
&+\left|\{y \in Q_j: (c_\alpha, y) > 0\}\right| - \left|\{y \in Q_j: (c_\alpha, y) \leq 0\}\right| = k_j - k_j + \left|\{y \in Q_j: (c_\alpha, y) > 0\}\right| - \\
&-\left|\{y \in Q_j: (c_\alpha, y) \leq 0\}\right| > 0.
\end{aligned}
$$

For $c_\alpha/\|c_\alpha\| \in O_{\varepsilon 1}(-c_j)$ the given inequality is verified in the same way. So, Q is the committee.

Let us consider points $d_1, \dots, d_r$. Since among $d_1, \dots, d_r, c_1, \dots, c_m$ there no null or collinear ones, then similarly to the existence theorem in case of a finite system [4], there will be found a set $P_1 = \{h_1, h_2, \dots, h_{2r-1}, h_{2r}\}$ such that

1) $\left(h_{2i-1,} d_1\right) > 0, (h_{2i}, d_i) > 0, \left(h_{2j}, d_i\right)\left(h_{2j-1}, d_i\right) < 0 \quad i \neq j, i, j \in \overline{1, r}$;
2) $(h_{2i-1}, c_j) < 0 \quad i, j \in \overline{1, r}$.

Since all inequalities are strict, they still stand when substituting $d_1, \dots, d_r, c_1, \dots, c_m$ for the points from the corresponding ε-neighbourhoods (at sufficiently small ε). We will suppose that $\varepsilon < \varepsilon_1$. According to lemma 1, having reproduced every pair h_{2i-1}, h_{2i} sufficient number of times and obtained a new set *P*, we will achieve that $Q \cup P$ will be a subsystem committee.

$(c_\alpha, x) > 0$, where $c_\alpha/\|c_\alpha\|$ lies in ε-neighbourhood of some limit point. It happens due to the fact that when voting close to points $d_1, \dots, d_r$ the impact of *P* prevails, and when voting close to $c_1, -c_1 \dots, c_m, -c_m$the members of *P* set compensate each other in pairs.

Let us consider those $c_\alpha$which do not belong to this system, their finite number. Let us denote all these points $f_1, f_2, \dots f_k$. Let us apply lemma 2 for those f_i where $|\{y \in Q \cup P: (f_i, y) > 0\}| - |\{y \in Q \cup P: (f_i, y) \leq 0\}| = \lambda_i \leq 0$.

Having obtained W set according to lemma 2 and having reproduced i-pair $-\lambda_i + 1$ times in it, we will obtain W'set and thus we will construct the required committee $Q \cup P \cup W'$ of our system. The theorem is proved.

The given theorem shows that in case of a finite number of limit points of set $\{c_\alpha/\|c_\alpha\|\}$ the problem of finding a system committee reduces to the problem of finding independent committees close to the limit point. The only restriction put to the certain committee is that the limit points (except for the one in the ε-neighbourhood of which the given committee is separating) shall not be laid in the hyperplanes forming the set. In fact, the restriction is not too strict nevertheless it is essential. Moreover, as the following example shows, when the number of space dimensions is more than two, even the fact of local consistency of the system close to every limit point does not secure that the whole system will be committee solvable.

Example 1.1. Let us consider the problem of A and B set separation by linear functionals in R^3 space. Suppose there exist four semispheres tangent to planes $x_1 = 0$ in points $c_1 = [0,1,1]$ and $c_2 = [0,-1,1]$. Note that vectors c_1 and c_2 are not collinear. Suppose sets A and B are isolated points on the semispheres condensing when approaching c_1 and c_2. Thus, the set of limit points $A \cup B$ is two-element. Near each of them there exists a single separating hyperplane whereas there exists no committee separating sets A and B since the hyperplanes (it is easily seen that they are to be present as the members in any committee) mutually destroy each other because they always vote in the contrary way [6].

Theorem 1.2. Suppose system $(c_\alpha, x) > 0$ $(\alpha \in M)$ satisfies the following conditions:

1) Among vectors c_α there exist no null or opposite ones.
2) Any consistent subsystem is a part of a certain maximum consistent subsystem.
3) The set of all μ-subsystems is finite.

Then the system is committee solvable.

Proof. Suppose $h_1, h_2, \dots, h_m$ represent some solutions of all diverse μ-subsystems. Let us assign an ordered set of m numbers $e_\alpha = \{z_1, z_2, \dots, z_m\}$ to every vector c_α where

$$z_i = \begin{cases} 1, if\ (c_\alpha, h_i) > 0, \\ -1, if (c_\alpha, h_i) \le 0. \end{cases}$$

Two vectors c_α and c_β will be assumed as equivalent if $e_\alpha = e_\beta$. Thus, the whole set $\{c_\alpha\}$ is divided into a finite number of classes: $\{c_\alpha\} = C_1 \cup C_2 \cup \dots \cup C_k$. One class includes the vectors indiscernible in terms of maximum consistent subsystems. Let us take one arbitrary representative from every class. Let us denote them as $c_1, c_2, \dots, c_k$. Among vectors $c_1, c_2, \dots, c_k$ there exist no null or opposite ones therefore there will be found a committee $K = \{x^1, x^2, \dots, x^q\}$ of subsystem $(c_j, x) > 0$ $j \in \overline{1,k}$. For every $i \in \overline{1,q}$ we assign a set $J_i = \{j \in \overline{1,k} : (c_j, x^i) > 0\}$.

Let us consider a subsystem $(c_j, x^i) > 0$. Since this subsystem is consistent then according to condition 2) there exists a maximum consistent subsystem the index of which has J_i. Suppose y_i is a certain solution of this μ-subsystem. We can assume that $y_i \in \{h_1, h_2, \dots, h_m\}$. But then the set $\{y_1, y_2, \dots, y_k\}$ is a committee of the initial system because for any point of the same class the members of the given set vote equally. The theorem is proved [3].

Note that when analyzing committee solvability of a finite system of linear inequalities it is sufficient to consider committees made up of solutions of some of the maximum consistent subsystems of the system. It happens due to the fact that the index of every consistent subsystem is included in the index of some μ-subsystem. As a result the assertion [5] is proved that states that among minimum system committees $(c_j, x) > 0$ $(j \in N_m)$ there exists the one where all members constitute the solutions of some μ-subsystems. The given and its basic assertions occur incorrect in case of infinite system of linear inequalities in spaces with the number dimensions is more than two. For this very reason condition 2) in the theorem is essential. The mentioned above confirms the following example.

Example 1.2. Consider a system:

$$0x_1 + 2x_2 + x_3 > 0,$$
$$(1/n^2)x_1 + (1/n)x_2 + x_3 < 0,$$
$$-(1/n^2)x_1 + (1/n)x_2 + x_3 > 0, n = 1, 2, \dots$$

Every finite subsystem of the given system is consistent. The only μ-subsystem is:

$(1/n^2)x_1 + (1/n)x_2 + x_3 < 0$,

$-(1/n^2)x_1 + (1/n)x_2 + x_3 > 0, n = 1, 2, ...$

i.e. there exists no maximum (in sense of including) inconsistent subsystem that could include the first inequality. One of the minimum committees is the set: $y^1 = [-1, 0, 0], y^2 = [-1, 0, 2], y^3 = [0, 1, -1.5]$ while $y^2, y^3$are not the solutions of μ-subsystems [6].

2. Committees of systems of linear heterogeneous inequalities

In this part we will consider a question of existence of a committee of a linear inequality system $(c_\alpha, x) > b_\alpha$ $(\alpha \in M)$. Note that it becomes necessary to consider systems of heterogeneous linear inequalities separately in terms of their committee solvability due to the infinite number of inequalities in the system since in case of a finite system the following assertion is correct.

Assertion [4, 6]. Existence of a collective solution of a homogeneous system $(c_j, x) > 0$ $(j \in N_m)$ entails existence of a collective solution of a heterogeneous system $(c_j, x) > b_j$ $(j \in N_m)$ with arbitrary numbers $b_1, b_2, ..., b_m$. It is easy to bring the example showing that in case of infinite systems the given assertion is incorrect. Therefore the problem of the search of the necessary conditions for committee solvability of heterogeneous systems arises. We will assume that among vectors c_α there exist no null ones. Then holds

Theorem 2.1. For a committee of the system $(c_\alpha, x) > b_\alpha$ $(\alpha \in M)$ to exist it is necessary that the set $\{b_\alpha/\|c_\alpha\|\}$ is limited from above i.e. $\exists L > 0: b_\alpha/\|c_\alpha\| < L$ for all $\alpha \in M$ [4].

Proof. By contradiction. Suppose there is a set $Q = \{x^1, x^2, ..., x^q\}$ which is a system committee, the set $\{b_\alpha/\|c_\alpha\|\}$ not being limited from above. Therefore there will be found a sequence $b_n/\|c_n\| \to +\infty$. Suppose $K = \max\{\|x^i\|\}$. Then we get $|(c_\alpha/\|c_\alpha\|, x^i)| \le \|x^i\| \le K$. Therefore starting from some number m with $n > m$

$(c_n/\|c_n\|, x^i) - b_n/\|c_n\| < 0$ $\forall i \in \{1, 2, ... q\}$.

It contradicts the fact that q is a committee. The theorem is proved.

As a rule the problem of finding committee constructions for homogeneous inequality systems is easier than that for the relative heterogeneous ones. In some cases it can be applied. For the systems over R^2 space the following theorem holds true.

Theorem 2.2. Suppose the following conditions are met for a system of heterogeneous linear inequalities $c_\alpha, x) > b_\alpha$ $(\alpha \in M)$ over R^2:

1) $\exists N \in R: b_\alpha/\|c_\alpha\| < N$;

2) If $c, -c \in \overline{\{c_\alpha/\|c_\alpha\|\}}$, then there exist such numbers $\varepsilon > 0, L > 0$ that $b_\alpha/|(c_\alpha, c)| < L$ where $\alpha \in M(c, -c, \varepsilon)$, $(c, c^\perp) = 0$, $c^\perp \neq 0$.

Then the existence of a committee of a homogeneous system $(c_\alpha, x) > 0$ entails the existence of a committee of a system $(c_\alpha, x) > b_\alpha$ $(\alpha \in M)$ [5].

Proof. Without restricting the generality it can be assumed that among c_α there exist no like-directed vectors since otherwise out of all inequalities $(c_\alpha/\|c_\alpha\|, x) > b_\alpha/\|c_\alpha\|$ with the identical left part it is sufficient to leave only one which has the maximum right part without breaking the line of argument.

Suppose K is a committee of a system $(c_\alpha, x) > 0$ made up from the solutions of different maximum consistent subsystems. Then all the points of K committee can be divided into two groups: $K = \{x^1, x^2, ..., x^q\} \cup \{y^1, y^2, ..., y^k\}$. The first group includes those vectors x^1 for which will be found $c, -c \in \overline{\{c_\alpha/\|c_\alpha\|\}}$ such that $(x^i, c) = 0$. The second group includes all other vectors y^i. Let us consider an arbitrary point $y^i \in \{y^1, y^2, ..., y^k\}$ and a line $(y^i, x) = 0$. By definition at least one of the two points of intersection of the given line with the unit circle does not represent a limit point of the set $\{c_\alpha/\|c_\alpha\|\}$. Therefore turning vector y^i by a sufficiently small angle, we can achieve that none of the points of intersection represents a limit one, at the same time K will still be the committee of the system $(c_\alpha, x) > 0$. Then it can be easily seen that $\lambda_i = \inf(c_\alpha/\|c_\alpha\|, y^i) > 0$, where inf is taken for such c_α where $(c_\alpha, y^i) > 0$. Let us demonstrate that at sufficiently large $\lambda > 0 : (c_\alpha, y^i) > 0 \Rightarrow$ $\Rightarrow (c_\alpha, \lambda y^i) > b_\alpha$. Suppose $\lambda = \frac{N}{\min(\lambda_i)}$. Then we have $(c_\alpha, \lambda y^i) = \lambda\|c_\alpha\|(c_\alpha/\|c_\alpha\|, y^i) \ge$

$\geq \lambda\|c_\alpha\|\lambda_i = N\|c_\alpha\|\lambda_i/\min(\lambda_i) > b_\alpha$. Now let us consider an arbitrary point $x^i \in \{x^1, x^2, \ldots, x^q\}$. Then the line $(x^i, x) = 0$ intersects the unit circle in points *c* and –*c* which are the limit points of the set $\{c_\alpha/\|c_\alpha\|\}$. Let us find such a number $\mu_i > 0$ that for the arbitrary vector $c_\alpha/\|c_\alpha\|(\alpha \in M)$ falling within ε-neighbourhood of *c* and –*c* point $(c_\alpha, x^i) > 0 \Rightarrow (c_\alpha, \mu_i x^i) > b_\alpha$ will be true. Suppose $\mu_i = L\|c^\perp\|/\|x^i\|$. Then $(c_\alpha, \mu_i x^i) = \mu_i\|x^i\|(c_\alpha, x^i/\|x^i\|) = \mu_i\|x^i\||(c_\alpha, c^\perp/\|c^\perp\|)| > \mu_i\|x^i\|b_\alpha/(L\|c^\perp\|) = b_\alpha$. For vectors c_α not falling within ε-neighbourhood of points *c* and –*c* such that $(c_\alpha, x^i) > 0$ the following is true: $\nu_i = \inf(c_\alpha/\|c_\alpha\|, x^i) > 0$. Therefore, as it was shown above there will be found $\eta_i > 0$ such that for these vectors $(c_\alpha, x^i) > 0 \Rightarrow (c_\alpha, \eta_i x^i) > b_\alpha$. Now it is sufficient to set $\omega_i = \max\{\eta_i, \mu_i\}$ and we come to a final conclusion: for an arbitrary *z* member of *K* committee there will be found such a positive number λ_z that $(c_\alpha, z) > 0 \Rightarrow (c_\alpha, \lambda_z z) > b_\alpha$, then it means there exists a required committee of a heterogeneous system. The theorem is proved.

Conclusions

1. The concept of committee is applied successfully not only for finite inequality systems but for infinite ones as well.
2. Committee decision rules are the means of solution of the problems of infinite separated sets discrimination [1, 2].

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ЭКЗИСТЕНЦИАЛЬНЫЕ ВОПРОСЫ КОМИТЕТНЫХ КОНСТРУКЦИЙ. ЧАСТЬ II

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Рассматривается вопрос о существовании комитета системы линейных неравенств при дополнительных условиях. Большую часть статьи занимают результаты исследований Вл.Д. Мазурова и М.Ю. Хачая по комитетам систем линейных неравенств. Данная статья – непосредственное продолжение этих результатов. Даётся ответ на вопрос, каковы доказательства этих результатов в бесконечномерном случае. Это особенно трудный раздел проблемы.

Комитет системы алгебраических неравенств – упорядоченное множество решающих правил, на основании которого формируется окончательная процедура принятия решений.

Задача построения комитетов и их использование в экономике и технике актуальна, так как часто исходная их формулировка содержит противоречия и неформализованные разделы. Здесь рассматривается система однородных линейных неравенств с бесконечным множеством индексов. Множество решений может быть и пустым. Доказываются условия, при которых существует комитет этой системы. Из этой теоремы следует, что когда число предельных точек в левых частях неравенств конечно, то задача сводится к задаче построения независимых друг от друга комитетов. Приводится пример.

В настоящее время большое значение приобретает факторный анализ с подобными особенностями, и для них тоже подходит предлагаемый математический аппарат.

Эти же методы используются и в психологии, в том числе психологии бессознательного, изучение которой было инициировано К. Юнгом.

Ключевые слова: распознавание, система неравенств, разделение множеств.

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