

SIMULATION OF PLASTIC DEFORMATIONS IN METAL ROLLING

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Mathematical simulation of plastic deformations in rolling of porous materials involves a consequent combination of the general energy relationship in plasticity and the variation inequality expressing the principle of minimum of entire deformation energy. A real deformation state in a plastic zone beneath rolls and corresponding kinematic and dynamic conditions on the contact surface are considered as a limited one for the consequent approximate deformation states and found out by the method of approximated approach. Any realization of this kind of method on personal computers requires a rational construction of a kinematic admissible velocity field in the spatial domain on plastic flow. Using the ordinary propositions and assumptions it became possible to construct one of the simple spatial kinematic admissible velocity field in the plastic zone beneath rolls. On the base of a big amount of experimental data a new hypothesis and analytic function describing the density distribution along the plastic zone during the rolling process have been proposed. On the base of the consequent approach in which the general energetic relationship and the variation inequality are interrelated it has been able to find out the geometric, kinematic and dynamic characteristics of plastic deformation states in rolling of porous materials.

Keywords: simulation, deformation, rolling, velocity field, plastic zone.

Model of plastic deformations in rolling of porous materials

A real deformation state in a plastic zone beneath rolls and corresponding kinematic and dynamic conditions on the contact surface are considered as a limited one for the consequent approximate deformation states and found out by the method of approximated approach [1–5]. In this process the real values of the variation parameters describing the deformation state, kinematic and dynamic conditions on the contact surface are calculated by the step-by-step using of the basic energetic relationship of plasticity [6–11]:

$$\int_V THdV + \sum_j \int_{\Omega_j} \tau_s |\Delta v_\tau|_j d\Omega - \int_{F_k} \bar{p} \bar{v}_k dF = 0 \quad (1)$$

and the variation inequality describing the principle of minimum of entire deformation energy [12–14]:

$$\int_V TH^* dV + \sum_j \int_{\Omega_j} \tau_s |\Delta v_\tau|_j d\Omega - \int_{F_k} \bar{p} \bar{v}_k^* dF \geq 0, \quad (2)$$

where an asterisk marks is for the values relating to a kinematic admissible deformation state in the plastic zone; H – the intensity of the shear

strain rates; T – the intensity of the shear strains ($T = \tau_s$ – in the state of ideal plasticity); $\bar{p} = \{p_x; p_y; p_z\}$ – a vector of the surface unit pressure and its components; $\bar{v} = \{v_x; v_y; v_z\}$ – a vector-velocity of a particle displacement in the deformation zone V ; $|\Delta v_\tau|_j$ – a leap of the tangent component of a velocity on a discontinuous surface Ω_j ; F_k – the contact surface. In constructing the mathematical model of the rolling process the following assumptions are adopted:

1) any plastic deformation state is described in accordance by the plane cross sections hypothesis [15], when

$$\sigma_x = \sigma_x(x), v_x = v_x(x), \rho_x = \rho_x(x); \quad (3)$$

2) an increasing of porous material density $d\rho$ on any cross-section $x \in [0; l]$ of the plastic zone is proportional to a product of a relative degree of deformation $dh/h(x)$ on this cross-section, of an accumulative logarithmic deformation $\ln(h_0/h(x))$ and at the same time is inversely proportional to strip density at the same cross-section of the deformation zone, that is

$$d\rho = A \left[\ln \frac{h_0}{h(x)} \right]^m \frac{dh/h(x)}{[\rho(x)]^{n-1}}, \quad (4)$$

where A is a proportional coefficient; m and n ($m \geq 1$, $n \geq 2$) are parameters characterizing

the rate of porous strip compactness in the direction of rolling (Fig. 1).

Solving differential relationship (4) under the boundary conditions given:

$$\rho(x)|_{x=l} = \rho_0, \rho(x)|_{x=0} = \rho_1, \quad (5)$$

where $x = l$ is the equation of the enter plane and $x = 0$ is the equation of the exit plane of the deformation zone in rolling respectively, it is found out the following analytic function describing porous material density distribution along the plastic deformation zone (Fig. 2):

$$\rho(x) = \rho_0 \sqrt[n]{1 + \left[\left(\frac{\rho_1}{\rho_0} \right)^n - 1 \right] \left\{ \frac{\ln h_0/h(x)^{m+1}}{\ln h_0/h_1} \right\}}; \quad (6)$$

3) density of porous strip and any kinematic-admissible deformation state are interrelated by according to the constant mass law which is written as follows

$$\operatorname{div}(\rho \bar{v}) = \frac{d}{dx}(\rho v_x) + \rho(x) \left[\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right] = 0, \quad (7)$$

where the assumptions mentioned above are taking into account;

4) on the contact surface $z = h(x) = R + h_1 - \sqrt{R^2 - x^2}$, $x \in [0; l]$, there is a relative movement and an intensive force vector of friction f is defined as following:

$$\bar{\tau} = f p(x) \frac{\Delta \bar{v}_k}{|\Delta \bar{v}_k|}, \quad (8)$$

where f is a friction coefficient; $p = p(x)$ is a function describing the changing of intensive normal pressure on the contact surface along the direction of rolling; is a velocity-vector of relative movement on the contact surface.

An initial deformation state is chosen as that concerning to rolling process without broadening. Using basic energetic relationship (1) adopted to an elementary volume as a thin cross-section layer of the deformation zone we have got a differential equation of the distribution of normal

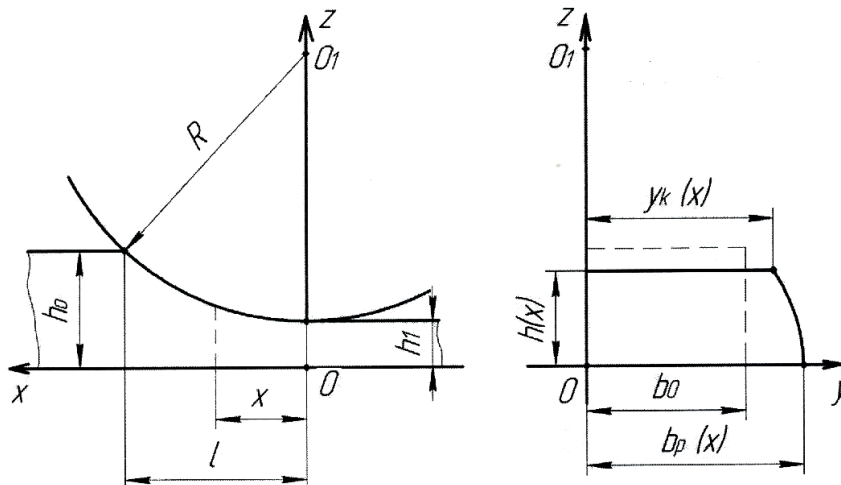


Fig. 1. Scheme of rolling process

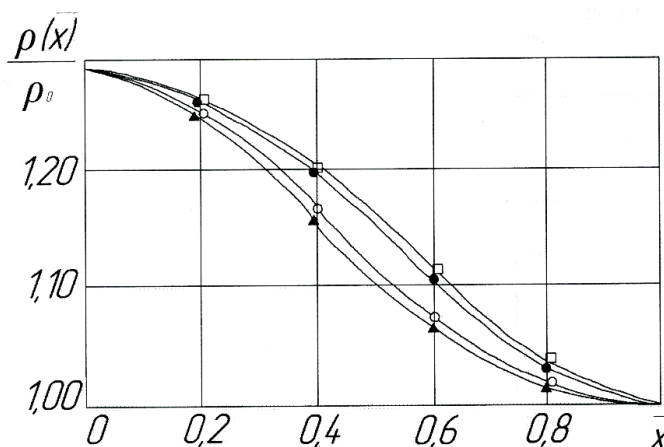


Fig. 2. Density curves of porous strip along the deformation zone ($\bar{x} = x/l$) when the ratio is $\frac{\rho_1}{\rho_0} = 1,29$: \bullet - $m = 1, n = 1$; \blacktriangle - $m = 2, n = 2$; \square - $m = 1, n = 3$; \circ - $m = 2, n = 3$

intensive pressure on the contact surface. Solving this equation with taking into account the characteristics of the initial deformation state there found out the corresponding dynamic ($p = p(x)$) and kinematic (a value of critical angle γ) conditions on the contact surface. Then fixing on these dynamic and kinematic conditions on the base of variation inequality (2) there are found out characteristics of the first deformation approach state. Repeating the procedure mentioned above there are defined corresponding kinematic and dynamic conditions on the contact surface. Analogously there are calculated the following approaches of deformation states. An exit of this iteration process is done by means of reaching the accuracy given before for one of the important technologic parameters.

A kinematically admissible velocity field in the deformation zone

Let's consider the deformation zone in rolling of porous strip with a rectangular cross-section area in two rolls when a changing of broadening is not uniform along a height of rolling porous strip. Due to symmetry of plastic flow along the height and width of the deformation zone the analytic describing of the rolling process is considered for 1/4 part of the deformation zone (Fig. 1). A kinematically admissible velocity field and corresponding deformation state of plastic zone are constructed on the assumptions mentioned above and the following addition hypotheses:

5) there is a relative movement of material particles on the contact surface:

$$v_n = v_x(x) \frac{dh}{dx} - v_z[x, y, h(x)] = 0; \quad (9)$$

6) there are the kinematic conditions on symmetric planes:

$$v_y(x, 0, z) = 0; \quad v_z(x, y, 0) = 0; \quad (10)$$

7) the components ξ_y and ξ_z of the deformation rates at any point of the plastic zone are interrelated as follows:

$$\frac{\xi_y}{\xi_z} = k(x, z, a_0, a_1, a_2), \quad (11)$$

where

$$k(x, z, a_0, a_1, a_2) = a_0 + a_1 \frac{z}{h(x)} + a_2 \frac{z^2}{h^2(x)},$$

a_0, a_1, a_2 – the variation coefficients the real values of which are found out as the result of solving the general variation problem of plasticity. Taking into account all of assumptions and hypotheses mentioned above it is found out the following functions describing the kinematically

admissible velocity field in the deformation zone beneath rolls:

$$\begin{cases} v_x(x) = \frac{\rho_1}{\rho(x)} v_1 \left[\frac{h_1}{h(x)} \right]^{\frac{1}{\theta}}; \\ v_y(x, y, z) = -\frac{1}{\rho(x)} \frac{k(x, z)}{1+k(x, z)} y \frac{d}{dx} (\rho v_x); \\ v_z(x, z) = -\frac{1}{\rho(x)} \frac{d}{dx} (\rho v_x) \int_0^z \frac{dt}{1+k(x, t)}, \end{cases} \quad (12)$$

where $\theta = \int_0^1 \frac{du}{1+a_0+a_1u+a_2u^2}$.

Relying on the values and relationship of the variation coefficients a_0, a_1, a_2 and the variation parameters n, m, ρ_1 in formula (6) the different variants of kinematic admissible states in the plastic deformation zone are constructed.

Kinematically admissible trajectories of particles displacement in the deformation zone

As is known on the base of functions (12) and the differential equations of trajectories, namely

$$\frac{dy}{dx} = \frac{v_y(x, y, z)}{v_x(x)}, \quad \frac{dz}{dx} = \frac{v_z(x, z)}{v_x(x)}, \quad (13)$$

where x is an independent variable, we are able to calculate on any computer by means of one of numerical methods the kinematically admissible trajectories of particles displacement in the plastic deformation zone in rolling. Let's point out two simple partial cases of solving of system (13).

SOLUTION 1. Let's find out trajectories of particles displacement on the horizontal symmetry plane $z = 0$. At this case differential system of equations (13) is transformed as follows:

$$\frac{dy}{dx} = -\frac{a_0}{1+a_0} \frac{y}{\rho v_x} \frac{d}{dx} (\rho v_x); \quad \frac{dz}{dx} = 0. \quad (14)$$

Integrating of system (14) gives the following equations of trajectories on the plane $z = 0$:

$$y(x) = y_0 \left[\frac{\rho(x) v_x(x)}{\rho_0 v_0} \right]^{-\frac{a_0}{1+a_0}}; \quad z(x) = 0, \quad (15)$$

where y_0 is an y -coordination of a particle on the enter cross-section plane $x = 1$.

SOLUTION 2. Let's find out trajectories of particles on the contact surface $z = h(x)$. Reasoning analogously, we have got the following equations:

$$y(x) = y_0 \left[\frac{h(x)}{h_0} \right]^{\frac{B}{\theta}}; \quad z(x) = h(x), \quad (16)$$

where $B = \frac{a_0+a_1+a_2}{1+a_0+a_1+a_2}$.

Analyses of the kinematically admissible model of particles displacement shows that the values of variation coefficients and parameters and as well their relationship give the following two cases on the contact surface:

- 1) there is plastic flow along the Y -axis if $a_0 + a_1 + a_2 \neq 0$;
- 2) there is no plastic flow along the Y -axis if $a_0 + a_1 + a_2 = 0$.

Results

1. It is propounded a new hypothesis of density distribution of porous strip along the plastic zone in rolling and it is got corresponding analytic function.

2. In case of simple rolling process, it is constructed spatial kinematically admissible velocity field (12) that is not uniform along the height (the Z -axis) and the width (the Y -axis) of the plastic deformation zone.

3. It is worked out the method of approxi-

mated approaches inside of which the basic energetic relationship of plasticity (1) and the variation inequality (2) expressing the principle of minimum of entire deformation energy are interrelated.

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МОДЕЛИРОВАНИЕ ПЛАСТИЧЕСКИХ ДЕФОРМАЦИЙ ПРИ ПРОКАТКЕ

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Математическое моделирование пластических деформаций при прокатке пористых материалов состоит из последовательного сочетания общих энергетических соотношений пластичности и неравенства вариаций, выражающих принцип минимума всей энергии деформации. Реальное напряженно-деформированное состояние в пластической зоне под валками и соответствующие кинематические и динамические условия на поверхности контакта рассматриваются как граничные для последующих приближенных состояний деформации и обнаруживаются методом приближенного подхода. Любая реализация этого метода на персональных компьютерах требует рационального построения кинематического поля допустимой скорости в пространственной области на пластическом течении. Используя обычные предложения и допущения, стало возможным построить одно из простых пространственных кинематических полей допустимой скорости в пластической зоне под валками. На основе большого количества экспериментальных данных была предложена новая гипотеза и аналитическая функция, описывающая распределение плотности вдоль пластической зоны в процессе прокатки. На основе последовательного подхода, в котором общие энергетические отношения и вариационное неравенство взаимосвязаны, удалось выяснить геометрические, кинематические и динамические характеристики пластических деформационных состояний при прокатке пористых материалов.

Ключевые слова: моделирование, деформация, прокатка, поле скоростей, пластическая зона.

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