

# COMPUTER MODELING OF HOT ISOSTATIC PRESSING PROCESS OF POROUS BLANK

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Existing mathematical models for the process of HIP porous blanks are based on the solution of approximately differential equations of the equilibrium of quasi-continuous medium. The continual approach is used for computer modeling of the compressible body HIP process. Partial differential equations of motion for a quasi-continuous packing medium and physical equations for a viscous-plastic isotropic porous material subjected to work hardening are taken as the basis for the mathematical model by means of these simultaneous equations. Besides the equations of motion and the rheological equation, the equation of continuity deformation and the equation of heat flow are used. Numerical calculations for additional packing of a high-speed steel powder billet preliminary pressed in the hydrostat are performed. Numerical calculation of the problem for hardening the cylindrical high-speed steel billet with a mild steel shell is done using Lagrange's method by means of the difference scheme of continuous calculation of the Wilkins' type. Computer modeling allows to control the process of hardening and changing the form of a porous body during the HIP process.

*Keywords:* continuum; mathematical model; powder blank; different equation; rheology.

## Introduction

Hot isostatic pressing (HIP) is one of the most suitable techniques for producing the high-speed steel cemented carbide, superalloys, soft ferrites and composites at a simultaneous application of high-pressure and high temperature [1–3]. The governing of this advanced process is performed, as a rule, by computer. Therefore, development of more perfect mathematical models of the process of plastic deformation is the most important task of further improving the HIP technology and increasing the quality of new materials.

Two basic approaches to the construction of the mathematical model adequately describing the HIP process are generally used. The first method, a discrete one, is developed in the works [4–8]. Here, they consider different mechanisms of packing at HIP such, as plastic deformation, creeping, and diffusion described with kinetic equations. In the article [8] a porous material is modeled as a sphere with the spherical pore subjected to creeping deformation under isostatic loading conditions. However, introduction of the central symmetry considerably delays the packing process. It should be noted that there is a somewhat difficulty in application of the first method when modeling the HIP of workpieces having a complex configuration and shells which distort the uniformity of a stress-strain state of a porous body.

The second method, a continual one, is developed in [9–17]. Here, a porous body is considered as a complex packed medium with an irreversible volumetric strain. This allows to solve multidimensional problems of elasto-viscous-plastic flow of a porous body and a shell. In the work [17] the analyses of HIP

technological modes based on the application of both the first and the second methods are given.

We have developed the continual approach. Unlike the works of other authors, the present investigation deals with the differential equations of motion introduced into the mathematical model instead of equilibrium equations. This allows to employ Lagrange's method in numerical realization of the model by computer. The equations, in a special case, are written in the cylindrical coordinate system.

## 1. Basic equations for computer modeling

The HIP process, in a general case, is a three-dimensional non-stationary non-isothermal problem. Following the works [15–17] we have constructed simultaneous equations of motion describing the flow of a viscous-plastic isotropic compressible material subjected to strain hardening in the right-hand rectangular coordinate system  $x_i$ . Denote in the equations:  $\rho$  is the density,  $t$  is the temperature,  $\tau$  is the time,  $v_i$  is the component of the flow velocity vector,  $\sigma_{ij}$ ,  $s_{ij}$ ,  $\xi_{ij}$  are the components of stress tensors, a stress deviator, and strain rates. The basic equations have a form:

– the equation of motion

$$\rho \frac{dv_i}{d\tau} = \sigma_{ij,j}; \quad (1)$$

– the equation of continuity

$$\frac{1}{\rho} \frac{d\rho}{d\tau} + v_{i,i} = 0; \quad (2)$$

– the equation of heat conductivity

$$-c\rho \frac{dt}{d\tau} = (\lambda \text{ grad } t)_{i,i} + v\sigma_{ij}\xi_{ij}, \quad (3)$$

where  $c$  is the specific heat,  $\lambda = \lambda(t, \rho)$  is the coeffi-

cient of heat conductivity. The differentiation according to time is done along the path of motion of a material particle.

The components of the velocity vector and those of the deformation rate are determined with the known relations:

$$\frac{dx_i}{d\tau} = v_i, \quad (4)$$

$$\xi_{ij} = \frac{1}{2}(v_{i,i} + v_{j,i}). \quad (5)$$

The stress tensor, as usual, consists of a spherical constituent and a deviator:

$$\sigma_{ij} = \sigma\delta_{ij} + s_{ij}, \quad (6)$$

where  $\sigma$  is the mean normal stress, and  $\delta_{ij}$  is Kronecker's symbol. The equations (1)–(6) are completed with the rheological equations:

$$s_{ij} = 2\mu(\xi_{ij} - \frac{1}{3}\xi_{ij}\delta_{ij}), \quad (7)$$

$$\sigma = 3k\xi, \quad (8)$$

where  $\xi$  is the rate of relative change of volume,  $\mu$ ,  $k$  are coefficients of shear and volumetric viscosity dependent upon density and temperature.

The loading surface, in general case, has the form:

$$F(T, \sigma, \sigma_s, q_1, \dots, q_n) = 0, \quad (9)$$

where  $T = \sqrt{s_{ij}s_{ij}}$  is the intensity of tangential stresses,  $\sigma_s$  is the yield stress of a solid phase of the material and it is the temperature function,  $q_i$  are work hardening parameters.

## 2. Mathematical model of HIP process

Consider the mathematical model of the HIP isothermal process for a cylindrical porous body with the radius  $R$  and the height  $H$  within a shell. The body surface is subjected to the pressure of gas  $p = p_G(\tau)$  at a stationary temperature during the pressing cycle. The simultaneous equations (1)–(9) describing the viscous-plastic flow of a cylindrical porous compressible body is written as follows:

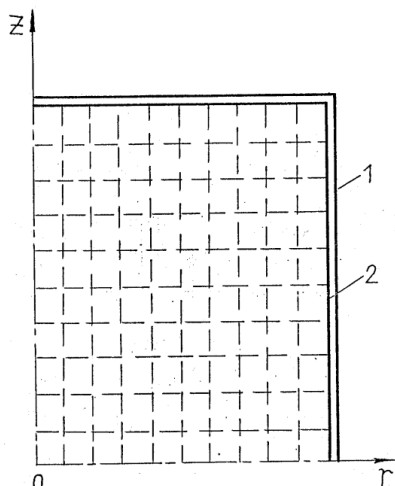


Fig. 1. Dynamics of a cylindrical porous body at HIP:  $p_G = 0-200$  MPa, at a velocity of the pressure increase  $w$ , equal to 10 MPa/s

$$\begin{aligned} \frac{1}{\rho} \frac{d\rho}{d\tau} + \frac{\partial v_z}{\partial z} + \frac{\partial v_r}{\partial r} &= -\frac{v_r}{r}, \\ \rho \frac{dv_r}{d\tau} &= \frac{\partial s_{rr}}{\partial r} + \frac{\partial s_{rz}}{\partial z} + \frac{s_{rr} - s_{\theta\theta}}{r} + \frac{\partial \sigma}{\partial r}, \\ \rho \frac{dv_z}{d\tau} &= \frac{\partial s_{rz}}{\partial r} + \frac{\partial s_{zz}}{\partial z} + \frac{s_{rz}}{r} + \frac{\partial \sigma}{\partial z}, \\ \frac{dr}{d\tau} &= v_r, \quad \frac{dz}{d\tau} = v_z, \\ s_{rr} &= 2\mu \left( \frac{\partial v_r}{\partial r} + \frac{1}{3\rho} \frac{d\rho}{d\tau} \right), \\ s_{zz} &= 2\mu \left( \frac{\partial v_z}{\partial z} + \frac{1}{3\rho} \frac{d\rho}{d\tau} \right), \\ s_{rz} &= \mu \left( \frac{\partial v_r}{\partial z} + \frac{dv_z}{dr} \right), \\ s_{\theta\theta} &= 2\mu \left( \frac{v_r}{r} + \frac{1}{3\rho} \frac{d\rho}{d\tau} \right), \\ \sigma &= \frac{k}{\rho} \frac{d\rho}{d\tau}, \\ T^2 &= s_{rr}^2 + s_{zz}^2 + s_{\theta\theta}^2 + 2s_{rz}^2, \\ F(T, \sigma, \sigma_s, q_i) &= 0. \end{aligned} \quad (10)$$

The behavior of the metallic shell of the porous cylinder corresponds to the elasto-plastic flow of the material [18]. The differential equations have to satisfy the following initial and boundary conditions:

$$\text{at } t = 0, \rho = \rho_0, s_{ij} = 0, \sigma = 0, \bar{v} = 0;$$

$$\text{at } t \geq 0 \text{ on the surface of the shell}$$

$$\sigma = -p_G(\tau), s_{rr} = s_{zz} = s_{\theta\theta} = s_{rz} = 0.$$

Within the boundary of the porous body and the shell the condition of continuity of the velocity vector  $\bar{v}$  is assigned.

Solution of the equations is done by means of different schemes of continuous calculations of Wilkins' type [18].

## 3. Results of numerical calculations

In numerical calculations the HIP process of a porous compressible high speed steel billet pressed in the hydrostat was modeled. The powder billet within

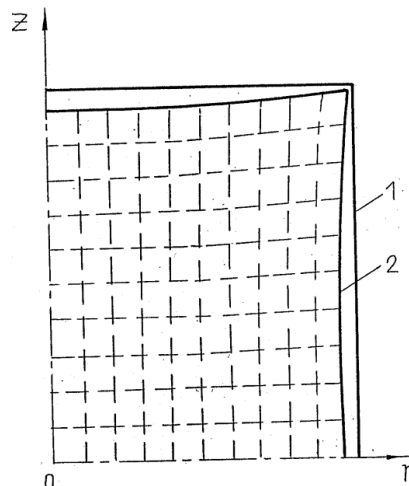


Fig. 2. Dynamics of a porous body at HIP:  $p_G = 0-200$  MPa,  $w = 500$  MPa/s,  $\rho_0 = 0,7\rho_c$

the mild steel shell was subjected to heating and then to hot gasostatic pressing.

The initial density of the porous body at HIP is assumed to be equal to  $0.93\rho_c$ , where  $\rho_c$  is the density of non-porous high speed steel, and the process temperature being as high as 1300, 1400 and 1500 °C. The dependence  $p_G(\tau)$  is taken as a piecewise linear one. The stress surface is assigned:

$$\frac{3}{2}T^2 + f_1(\bar{\rho})\sigma^2 = (f_2(\bar{\rho})f(\xi)\sigma_s)^2, \quad (11)$$

where  $\bar{\rho} = \frac{\rho}{\rho_c}$  is the relative density,

$$f_1(\bar{\rho}) = a_1(1 - \bar{\rho})^{n_1},$$

$$f_2(\bar{\rho}) = \bar{\rho}^{n_2},$$

$$f(\xi) = (a_2\xi^{n_3} + 1).$$

In Figs. 1 and 2 there are shown the cylindrical porous body at the initial instant (1) and final instant (2) in the HIP process as well as the difference grid on which the calculation is done. The coefficient values are taken from the works [7, 11].

## Conclusion

The HIP process of a porous compressible body is described with the mathematical model based on differential equations of motion for a quasi-continuous medium having an irreversible volumetric and shear deformation. The equations for a viscous-plastic isotropic porous hardening material are used as rheological ones. For the flows with an axial symmetry of the billet material after CIP the constitutive equations are written in the cylindrical coordinates. Numerical calculation of the problem for hardening the cylindrical high-speed steel billet within a mild steel shell is done using Lagrange's method by means of the difference scheme of continuous calculation of Wilkins' type.

Computer modeling allows to control the process of hardening and changing the form of a porous body at HIP in the course of time.

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## КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА ГОРЯЧЕГО ИЗОСТАТИЧЕСКОГО ПРЕССОВАНИЯ ПОРИСТЫХ ЗАГОТОВОК

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Известные математические модели процесса ГИП пористых заготовок основаны на решении приближенных дифференциальных уравнений равновесия квазисплошной среды. Использован континуальный подход построения компьютерной модели процесса горячего изостатического прессования (ГИП) порошковых заготовок. Представлена математическая модель процесса ГИП, основанная на решении дифференциальных уравнений движения квазисплошной уплотняемой среды, обладающей необратимой объемной и сдвиговой деформацией. В качестве реологических использованы уравнения вязкопластического течения изотропного упрочняющегося материала. Кроме уравнений движения и реологического уравнения при моделировании используются уравнение неразрывности деформации и уравнение теплопроводности. Численная реализация разработанной модели выполнена методом конечных элементов на примере ГИП цилиндрической заготовки из порошка быстрорежущей стали, полученной холодным прессованием в гидростате. Численный расчет по проблеме упрочнения цилиндрической заготовки из быстрорежущей стали, заключенной в оболочку из мягкой стали, выполнен по методу Лагранжа с помощью разностной схемы сквозного счета типа Уилкинса. Компьютерное моделирование позволяет контролировать процесс упрочнения и изменение формы пористого тела в течение процесса ГИП.

*Ключевые слова:* континуум; математическая модель; порошковая заготовка; дифференциальное уравнение; реология.

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