Математика

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ON THE EXACT SOLUTIONS TO CONFORMABLE EQUAL WIDTH WAVE EQUATION BY IMPROVED BERNOULLI SUB-EQUATION FUNCTION METHOD

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In this paper, we consider conformable equal width wave (EW) equation in order to construct its exact solutions. This equation plays an important role in physics and gives an interesting model to define change waves with weak nonlinearity. The aim of this paper is to present new exact solutions to conformable EW equation. For this purpose, we use an effective method called Improved Bernoulli Sub-Equation Function Method (IBSEFM). Based on the values of the solutions, the 2D and 3D graphs and contour surfaces are plotted with the aid of mathematics software. The obtained results confirm that IBSEFM is a powerful mathematical tool to solve nonlinear conformable partial equations arising in mathematical physics.

Keywords: improved Bernoulli sub-equation function method; conformable equal width wave equation; wave transformation.

Introduction

Fractional differential equations are the generalization of classical differential equations with integer order. So, in recent years, fractional differential equations become the field of physicists and mathematicians who investigate the expediency of such non-integer order derivatives in different areas of physics and mathematics. These equations became a useful tool for describing numerous nonlinear phenomena of physics such as heat conduction systems, nonlinear chaotic systems, viscoelasticity, plasma waves, acustic gravity waves, diffusion processes [1–3]. Many numerical and analytical methods were developed and successfully employed to solve these equations such as modified Kudryashov method [4], homotopy perturbation method [5], new extended direct algebraic method [6], fractional Riccati expansion method [7], modified extended tanh method [8].

During the last few years, a straightforward definition of conformable derivative is given [9]. The conformable derivative operator is compatible to many real-world problems and provides some properties of classical calculus such as derivative of quotient or product of two functions, the chain rule [10]. During the last few years, many of techniques applied to find exact solutions to conformable nonlinear partial differential equations [11–16].

In this paper, we consider the following conformable EW equation:

$$D_t^{\alpha} u + pD_x^{\alpha} u^2 - lD_{xxx}^{3\alpha} u = 0, \quad \alpha \in (0,1], \tag{1}$$

where p, l are real parameters, u is a function of independent variables. The operator D_t^{α} represents conformal derivative operator defined only for positive region of t [10]. Before consideration of the solution procedure, let us give some properties of conformable derivative.

The conformable derivative of order α with respect to the independent variable t is defined as [9]

$$D_t^{a}(y(t)) = \lim_{\tau\to 0} \frac{y(t+\tau t^{1-\alpha}) - y(t)}{\tau}, t > 0, \alpha \in (0,1]$$

for a function $y = y(t): [0,\infty) \to \mathbb{R}$.

Theorem 1.1. Assume that the order of the derivative is $\alpha \in (0,1]$ and suppose that $u = u(t)$ and $v = v(t)$ are $\alpha-$differentiable functions for all positive t. Then

i. $D_t^{a}(c_1 u + c_2 v) = c_1 D_t^{a}(u) + c_2 D_t^{a}(v)$ for $\forall\ c_1, c_2 \in \mathbb{R}$,

ii. $D_t^a(t^k)=kt^{k-a}\ \forall\ k\in\mathbb{R},$

iii. $D_t^a(\lambda)=0$ for each constant function $u(t)=\lambda,$

iv. $D_t^a(uv)=uD_t^a(v)+vD_t^a(u),$

v. $D_t^a\left(\frac{u}{v}\right)=\frac{vD_t^a(u)-uD_t^a(v)}{v^2},$

vi. $D_t^a(u)(t)=t^{1-\alpha}\frac{du}{dt}.$

Conformable differential operator satisfies some critical fundamental properties like the chain rule, Taylor series expansion and Laplace transform.

Theorem 1.2. Let $u=u(t)$ be an $\alpha-$conformable differentiable function and assume that v is a differentiable function. Then

$$D_t^a(u\circ v)(t)=t^{1-\alpha}v'(t)u'(v(t)).$$

The proofs of Theorems 1 and 2 are given in [17] and [9], respectively.

The rest part of the paper is organized as follows. In Section 1, descripton of the IBSEFM is given. In Section 2, the application of IBSEFM is mentioned. Finally, this study is completed by providing conclusions in the last section.

1. Description of the IBSEFM

In this section, we give the fundamental properties of the IBSEFM. This method is direct, significant, advanced algebraic method to establish reliable exact solutions for both nonlinear and nonlinear fractional partial differential equations [11,18–20].

Let us describe five main steps of the IBSEFM.

Step 1. Let us take into account the following conformable partial differantial equation of the form

$$P(v,D_t^{(\alpha)}v,D_x^{(\alpha)}v,D_{xt}^{(2\alpha)}v,...)=0, \tag{2}$$

where $D_t^{(\alpha)}$ is the conformable fractional derivate operator, $v(x,t)$ is an unknown function, P is a polynomial and its partial derivatives contain fractional derivatives. The aim is to convert conformable nonlinear partial differential equation with a suitable fractional transformation into the ordinary differantial equation. The wave transformation is

$$v(x,t)=V(\xi),\ \ \xi=\xi\left(x,t^\alpha\right). \tag{3}$$

Using the properties of conformable fractional derivate, we convert (2) into an ODE of the form

$$N(V,V',V'',...)=0. \tag{4}$$

If we integrate (4) term to term, we acquire integration constant(s), which can be determined later.

Step 2. We hypothesize that the solution to (4) can be represented as follows:

$$V(\xi)=\frac{\sum_{i=0}^{n}a_iF^i(\xi)}{\sum_{j=0}^{m}b_jF^j(\xi)}=\frac{a_0+a_1F(\xi)+a_2F^2(\xi)+...+a_nF^n(\xi)}{b_0+b_1F(\xi)+b_2F^2(\xi)+...+b_mF^m(\xi)}, \tag{5}$$

where $a_0,a_1,...,a_n$ and $b_0,b_1,...,b_m$ are coefficients which will be determined later. The numbers $m\neq 0,n\neq 0$ are chosen arbitrary constants to balance principle. Consider the Bernoulli differential equation of the form

$$F'(\xi)=\sigma F(\xi)+dF^M(\xi),d\neq 0,\sigma\neq 0,M\in\mathbb{R}/\{0,1,2\}, \tag{6}$$

where $F(\xi)$ is the solution to (6).

Step 3. The positive nonzero integers m,n,M are found by balance principle that is both nonlinear term and the highest order derivative term of (4).

Substituting (5), (6) in (2) gives the following equation of the polynomial $\Theta(F)$ of F:

$$\Theta(F(\xi))=\rho_sF(\xi)^s+...+\rho_1F(\xi)+\rho_0=0,$$

where $\rho_i, i = 0,...,s$ are coefficients and will be determined later.

Step 4. Equating all the coefficients of $\Theta(F(\xi))$ yields us an algebraic equation system

$$\rho_i = 0, i = 0,..,s.$$

Step 5. When we solve (4), we get the following two cases with respect to σ and d:

$$F(\xi) = \left[\frac{-de^{\sigma(\varepsilon-1)} + \varepsilon\sigma}{\sigma e^{\sigma(\varepsilon-1)\xi}}\right]^{\frac{1}{1-\varepsilon}}, \ d \neq \sigma, \tag{7}$$

$$F(\xi) = \left[\frac{(\varepsilon-1) + (\varepsilon+1)\tanh\left(\sigma(1-\varepsilon)\frac{\xi}{2}\right)}{1-\tanh\left(\sigma(1-\varepsilon)\frac{\xi}{2}\right)}\right], d = \sigma, \varepsilon \in \mathbb{R}. \tag{8}$$

Using a complete discrimination system of $F(\xi)$, we obtain the analytical solutions to (4) via mathematics software and categorize the exact solutions to (4). To achieve better results, we can plot two and three dimensional figures of analytical solutions by considering proper values of parameters.

2. Application of the IBSEFM

Take the travelling wave transformation

$$u(x,t) = U(\eta), \ \eta = q\frac{x^\alpha}{\alpha} + m\frac{t^\alpha}{\alpha}, \tag{9}$$

where q and m are nonzero constants to be determined. Then Equation (1) turns into the following ordinary differential equation:

$$mU + pqU^2 - qml^2U'' = 0. \tag{10}$$

When we apply the balance principle for the terms u^2 and u'', the result for m,n and M is

$$M + m = n + 1.$$

By balancing the order between the nonlinear term and highest order derivative in Equation (10), we obtain $M = 3, n = 3$ and $m = 1$, then we get

$$U(\eta) = \frac{a_0 + a_1F(\eta) + a_2F^2(\eta) + a_3F^3(\eta)}{b_0 + b_1F(\eta)} = \frac{\Upsilon(\eta)}{\Psi(\eta)},$$

$$U'(\eta) = \frac{\Upsilon'(\eta)\Psi(\eta) - \Upsilon(\eta)\Psi'(\eta)}{\Psi^2(\eta)}, \tag{11}$$

$$U''(\eta) = \frac{\Upsilon'(\eta)\Psi(\eta) - \Upsilon(\eta)\Psi'(\eta)}{\Psi^2(\eta)} - \frac{\left[\Upsilon(\eta)\Psi'(\eta)\right]'\Psi^2(\eta) - 2\Upsilon(\eta)[\Psi'(\eta)]^2\Psi(\eta)}{\Psi^4(\eta)}. \tag{12}$$

Substituting (3)–(11) in (10), we get the algeabraic equation system according to F:

$F^0 : 2pqa_0^2b_0 + ma_0b_0^2 = 0,$

$F : 4pqa_0a_1b_0 + ma_1b_0^2 - lmq^2\sigma^2a_1b_0^2 + 2pqa_0^2b_1 + 2ma_0b_0b_1 + lmq^2\sigma^2a_0b_0b_1 = 0,$

$F^2 : 2pqa_1^2b_0 + 4pqa_0a_2b_0 + ma_2b_0^2 - 4lmq^2\sigma^2a_2b_0^2 + 4pqa_0a_1b_1 + 2ma_1b_0b_1 + lmq^2\sigma^2a_1b_0b_1 +$
$+ ma_0b_1^2 - lmq^2\sigma^2a_0b_1^2 = 0,$

$F^3 : 4pqa_1a_2b_0 + 4pqa_0a_3b_0 - 4dlmq^2\sigma a_1b_0^2 + ma_3b_0^2 - 9lmq^2\sigma^2a_3b_0^2 + 2pqa_1^2b_1 + 4pqa_0a_2b_1 +$
$+4dlmq^2\sigma a_0b_0b_1 + 2ma_2b_0b_1 - 3lmq^2\sigma^2a_2b_0b_1 + ma_1b_1^2 = 0,$

$\vdots$

$F^9 : 2pqa_4^2b_0 + 4pqa_3a_5b_0 - 24d^2lmq^2a_4b_0^2 + 4pqa_3a_4b_1 + 4pqa_2a_5b_1 - 21d^2lmq^2a_3b_0b_1 -$
$-96dlmq^2\sigma a_5b_0b_1 - 3d^2lmq^2a_2b_1^2 - 24dlmq^2\sigma a_4b_1^2 = 0,$

$F^{10} : 4pqa_4a_5b_0 - 35d^2lmq^2a_5b_0^2 + 2pqa_4^2b_1 + 4pqa_3a_5b_1 - 37d^2lmq^2a_4b_0b_1 - 8d^2lmq^2a_3b_1^2 -$

$-40dlmq^2\sigma a_5 b_1^2 = 0,$

$F^{11}: 2pqa_5^2 b_0 + 4pqa_4 a_5 b_1 - 57d^2 lmq^2 a_5 b_0 b_1 - 15d^2 lmq^2 a_4 b_1^2 = 0,$

$F^{12}: 2pqa_5^2 b_1 - 24d^2 lmq^2 a_5 b_1^2 = 0.$

Then we solve the system of equations of F and, in each case, substitute the obtained coefficients to get the new solution(s) $u(x,t)$. Solving the system by Wolfram Mathematica software, we obtain the coefficients as follows.

Case 1. For $\sigma \neq d$,

$$a_0 = \frac{\sigma^2 a_4}{6d^2}; a_1 = \frac{\sigma^2 a_5}{6d^2}; a_2 = \frac{\sigma a_4}{d}; a_3 = \frac{\sigma a_5}{d}; b_1 = \frac{a_5 b_0}{a_4}; m = -\frac{pq\sigma^2 a_4}{3d^2 b_0}; l = -\frac{1}{4q^2\sigma^2}. \quad (13)$$

where $d, l, q, \sigma, a_4, b_0 \neq 0$.

Putting (13) along with (3)–(11) in (7), we acquire the exponential function solution to the EW equation as follows:

$$u_1(x,t) = \frac{\sigma^2\left(\frac{1}{d^2} + \frac{6}{\left(d - e^{\frac{2p\sigma\left(3t^\alpha + \frac{px^\alpha\sigma^2 a_4}{d^2 b_0}\right)}{3\alpha}}\right)^2 \varepsilon\sigma} - \frac{6}{d\left(d - e^{\frac{2p\sigma\left(3t^\alpha + \frac{px^\alpha\sigma^2 a_4}{d^2 b_0}\right)}{3\alpha}}\right)\varepsilon\sigma}\right)a_4}{6b_0}.$$

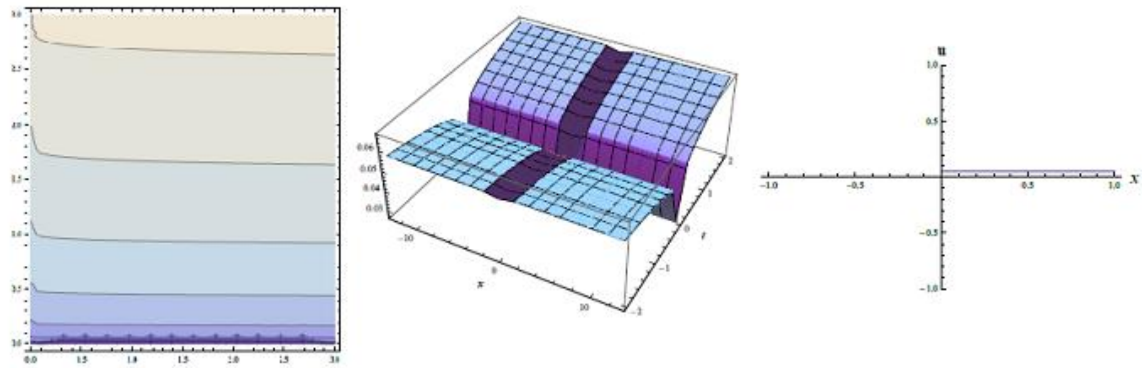

Fig. 1. The contourplot, 3D and 2D graphs of $u_1(x,t)$ by considering the values $\alpha = 0{,}1$; $\varepsilon = 0{,}2$; $\sigma = 0{,}1$; $d = 0{,}3$; $p = 0{,}5$; $q = 0{,}6$; $b_0 = 1$; $a_4 = 1$; $-13 < x < 13$, $-2 < t < 2$ **for 3D surface,** $0 < x < 3$, $0 < t < 3$ **for contourplot and** $-1 < x < 1$; $t = 0{,}2$ **for 2D**

Case 2. For $\sigma \neq d$,

$$a_0 = -\frac{a_4}{24d^2 lq^2}; a_1 = -\frac{a_5}{24d^2 lq^2}; a_2 = -\frac{ia_4}{2d\sqrt{l}q}; a_3 = -\frac{-ia_5}{2d\sqrt{l}q}; b_1 = \frac{a_5 b_0}{a_4}; p = -\frac{12d^2 lmqb_0}{a_4}; \sigma = -\frac{i}{2\sqrt{l}q}, \quad (14)$$

where $d, l, q, a_4, b_0 \neq 0$. Substituting (14) along with (3)–(11) in (7), we obtain solution to (1)

$$u_2(x,t) = \frac{\left(-l - \frac{12ide^{\frac{i\left(qt^\alpha + mx^\alpha\right)}{\sqrt{l}q\alpha}}\sqrt{l}q\varepsilon}{\left(-2ide^{\frac{it^\alpha}{\sqrt{l}\alpha}}\sqrt{l}q + e^{\frac{imx^\alpha}{\sqrt{l}q\alpha}}\varepsilon\right)^2}\right)a_4}{24d^2 lq^2 b_0}.$$

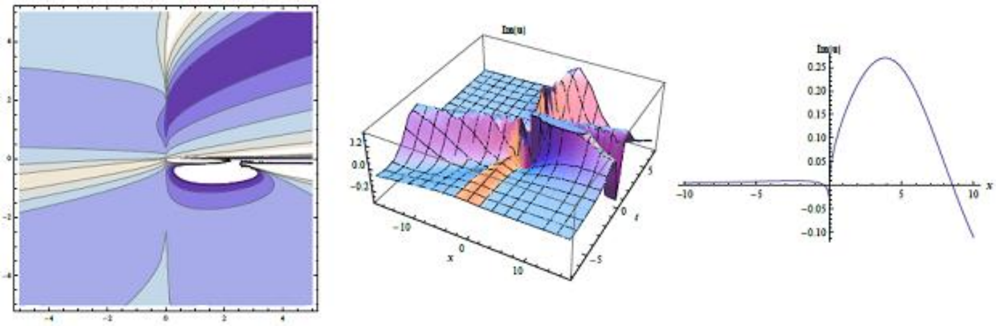

Fig. 2. The contourplot, 3D and 2D graphs of imaginary part of $u_2(x,t)$ by considering the values $\alpha = 0,4$; $\varepsilon = 0,1$; $\sigma = 0,2$; $d = 0,51$; $l = 0,5$; $m = 0,3$; $q = 0,73$; $b_0 = 0,6$; $a_4 = 0,2$; $-17 < x < 17$, $-7 < t < 7$ for 3D surface, $-5 < x < 5$, $-5 < t < 5$ for contourplot and $-10 < x < 10$; $t = 0,2$ for 2D

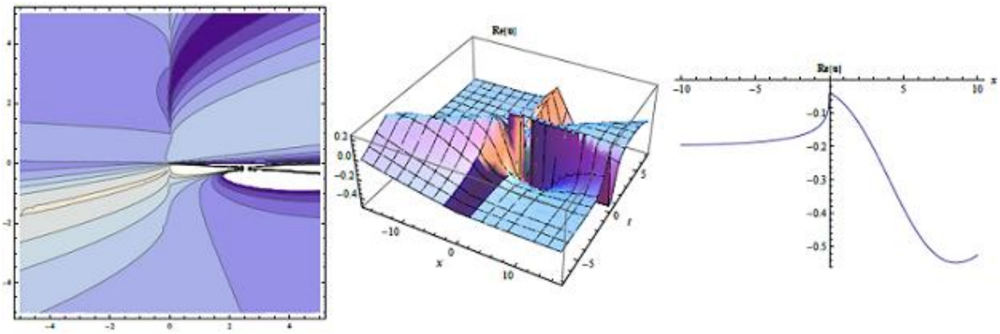

Fig. 3. The contourplot, 3D and 2D graphs of real part of $u_2(x,t)$ by considering the values $\alpha = 0,4$; $\varepsilon = 0,1$; $\sigma = 0,2$; $d = 0,51$; $l = 0,5$; $m = 0,3$; $q = 0,73$; $b_0 = 0,6$; $a_4 = 0,2$; $-17 < x < 17$, $-7 < t < 7$ for 3D surface, $-5 < x < 5$, $-5 < t < 5$ for contourplot and $-10 < x < 10$; $t = 0,2$ for 2D

Case 3. For $\sigma \neq d$,

$$a_0 = -\frac{mb_0}{2pq}; a_1 = -\frac{ma_5 b_0}{2pqa_4}; a_2 = -\frac{i\sqrt{3}\sqrt{m}\sqrt{a_4}\sqrt{b_0}}{\sqrt{p}\sqrt{q}}; a_3 = -\frac{i\sqrt{3}\sqrt{m}a_5\sqrt{b_0}}{\sqrt{p}\sqrt{q}\sqrt{a_4}}; b_1 = \frac{a_5 b_0}{a_4};$$
$$d = -\frac{\sqrt{p}\sqrt{a_4}}{2\sqrt{3}\sqrt{l}\sqrt{m}\sqrt{q}\sqrt{b_0}}; \sigma = -\frac{i}{2\sqrt{lq}}, \tag{15}$$

where $l, m, p, q, \sigma, a_4, b_0 \neq 0$. When we put (15) along with (3)–(11) in (7), and obtain the complex solution to (1) as follows:

$$u_3(x,t) = \frac{3m\left(e^{\frac{2it^{\alpha}}{\sqrt{l}\alpha}} pqa_4 + 4i\sqrt{3}e^{\frac{i\left(qt^{\alpha}+mx^{\alpha}\right)}{\sqrt{l}\alpha}}\sqrt{m}\sqrt{p}\sqrt{q}\varepsilon\sqrt{a_4}\sqrt{b_0} - 3e^{\frac{2imx^{\alpha}}{\sqrt{l}q\alpha}}m\varepsilon^2 b_0\right)}{2pq\left(i\sqrt{3}e^{\frac{it^{\alpha}}{\sqrt{l}\alpha}}\sqrt{p}\sqrt{q}\sqrt{a_4} + 3e^{\frac{2imx^{\alpha}}{\sqrt{l}q\alpha}}\sqrt{m}\varepsilon\sqrt{b_0}\right)^2}.$$

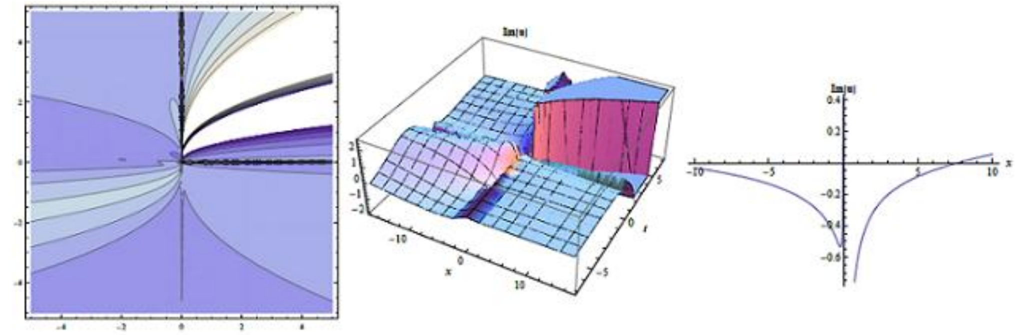

Fig. 4. The contourplot, 3D and 2D graphs of imaginary part of $u_3(x,t)$ by considering the values $\alpha = 0,1;$ $\varepsilon = 0,2;$ $\sigma = 0,3;$ $d = 0,62;$ $l = 0,6;$ $m = 0,2;$ $q = 0,47;$ $b_0 = 0,8;$ $a_4 = 0,5;$ $p = 0,3;$ $-17 < x < 17,$ $-7 < t < 7$ for 3D surface, $-5 < x < 5,$ $-5 < t < 5$ for contourplot and $-10 < x < 10;$ $t = 0,3$ for 2D

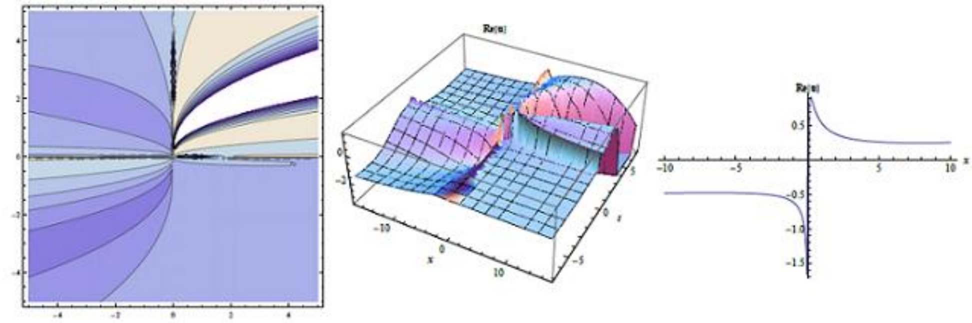

Fig. 5. The contourplot, 3D and 2D graphs of real part of $u_3(x,t)$ by considering the values $\alpha = 0,1;$ $\varepsilon = 0,2;$ $\sigma = 0,3;$ $d = 0,62;$ $l = 0,6;$ $m = 0,2;$ $q = 0,47;$ $b_0 = 0,8;$ $a_4 = 0,5;$ $p = 0,3;$ $-17 < x < 17,$ $-7 < t < 7$ for 3D surface, $-5 < x < 5,$ $-5 < t < 5$ for contourplot and $-10 < x < 10;$ $t = 0,3$ for 2D

Conclusion

In this paper, the IBSEFM method is applied for the conformable EW equation. Using a wave transformation, we convert the conformable differential equation into the ordinary differential equation, which can be solved according to the IBSEFM. By means of this method, exact solutions are obtained. The contourplot, 3D and 2D surfaces of all solutions obtained by IBSEFM under the suitable values of parameters are plotted to show the main characteristic physical properties of the solutions with the help of mathematics software. According to the results, one can see that the formats of travelling wave solutions in two and three dimensional surfaces are similar to the physical meaning of results.

The solutions are also solitary wave solutions. Also, it is clear that more steps are developed and better approximations are obtained. The conclusions show that the IBSEFM is simple, effective and powerful. Thus, in mathematical physics, it is applicable to solve other conformable partial differential equations. We claim that the IBSEFM method is practically well suited, since it can be adopted to a wide range of nonlinear differential equations. Eventually, this method is influential and suitable for solving other types of nonlinear differential equations in which the balance principle is satisfied.

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О ТОЧНОМ РЕШЕНИИ СОГЛАСОВАННОГО РАВНОМОЩНОГО ВОЛНОВОГО УРАВНЕНИЯ С ПОМОЩЬЮ УСОВЕРШЕНСТВОВАННОГО ФУНКЦИОНАЛЬНОГО МЕТОДА ПОД-УРАВНЕНИЯ БЕРНУЛЛИ

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В настоящей работе рассматривается согласованное равномощное волновое уравнение с целью нахождения его точного решения. Данное уравнение играет важную роль в физике и задает интересную модель определения изменяющихся волн со слабой нелинейностью. Целью работы является представление нового точного решения согласованного равномощного волнового уравнения. Для этого авторы используем эффективный метод, называемый усовершенствованным функциональным методом под-уравнения Бернулли (IBSEFM). На основе значений решений, двумерные и трехмерные графики и контурные поверхности строятся с привлечением математического программного обеспечения. Полученные результаты подтверждают, что IBSEFM является мощным математическим аппаратом для решения нелинейных согласованных уравнений в частных производных, возникающих в математической физике.

Ключевые слова: усовершенствованный функциональный метод под-уравнения Бернулли; согласованное равномощное волновое уравнение; волновое преобразование.

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