DOI: 10.14529/mmph230202

THE ANALYSIS AND PROCESSING OF INFORMATION FOR ONE STOCHASTIC SYSTEM OF THE SOBOLEV TYPE

K.V. Perevozchikova
South Ural State University, Chelyabinsk, Russian Federation
E-mail: perevozchikovakv@susu.ru

Abstract. This article is devoted to the analysis and processing of information for a stochastic model based on the equation of potential distribution in a crystalline semiconductor with the Nelson–Glicklich derivative and the Showalter–Sidorov initial condition. By semiconductors, we mean substances that have a finite electrical conductivity that rapidly increases with increasing temperature. It is assumed that the initial experimental data may be affected by random noise, which leads to the study of the stochastic model. An analysis of the stochastic model of the potential distribution in a crystalline semiconductor is given. Conditions under which there are step-by-step solutions of the model under study with the Showalter–Sidorov initial condition are found. Further, on the basis of the theoretical results, an algorithm for the numerical analysis of the system is given. Its implementation is presented in the form of a computational experiment, which is necessary for the further processing of information.

Keywords: stochastic model of potential distribution in a crystalline semiconductor; analysis and processing of information; the Nelson–Glicklich derivative; Sobolev type equations.

Introduction

As a rule, functioning of real physical processes is accompanied by the impact of hard-to-control perturbations. Under their influence, the study of systems is no longer possible in a deterministic case and, as a result, the transition to stochastic modeling takes place. Stochastic differential equations can be considered with various additive random processes. One of the classical ways to study stochastic models is the Ito–Stratonovich–Skorokhod method, which allows to move from differential equations to integral ones. At the moment, the method is extended to the infinite-dimensional situation [1], and various applications to classical models of mathematical physics are considered [2]. This approach has also been extended to degenerate Sobolev-type models [3].

Another direction in the study of stochastic models is the approach where “white noise” is considered as the Nelson–Gliklikh derivative $\overset{o}{\eta}$ of the Wiener process [4, 5]. An example would be a Shestakov–Sviridyuk model measuring device model [6]. This model is based on a Leontief type stochastic system. Note also that if η is a function then the Nelson–Glicklich derivative can be considered in the classical case. The idea of “white noise” in this theory, which existed in finite-dimensional spaces [7, 8] shows high efficiency. Therefore, later the principle was carried over to infinite-dimensional spaces [9, 10]. This approach allows us to transfer to the stochastic case the applied (well-known) methods of functional analysis in the deterministic case.

In this work, we present the analysis and processing of information for one stochastic system. To this end, first of all, we analyze the Sobolev-type stochastic model itself. Next, we find the conditions under which there exists a trajectory solution to the problem. This makes it possible to construct an algorithm for the numerical method and conduct a series of computational experiments. With the help of computational methods it is possible to process information. In the stochastic case, the mathematical model of the distribution of potential in a crystalline semiconductor has the form

$$\eta(s,t) = 0, (s,t) \in \partial D \times [0,T], \tag{1}$$

$$(\lambda - \Delta)\overset{o}{\eta} - a_1\Delta\eta - a_2 div(|\nabla\eta|^2 \nabla\eta) = 0, \tag{2}$$

with the weakened Showalter–Sidorov problem

$$\lim_{t\to 0+}(\lambda - \Delta)(\eta(t) - \eta_0) = 0, \quad s \in D. \tag{3}$$

Here the function $\eta = \eta(s,t)$ is the potential electric field, the parameters $\lambda \in \mathbb{R}, a_1, a_2 \in \mathbb{R}$, and $D \subset \mathbb{R}^n$ is a bounded domain and ∂D of the class C^∞. The mathematical interpretation of the model is presented in [11]. Note that in the stochastic model η_0 is a random influence. Problem (1), (2) can be reduced to the stochastic equation

$$L\overset{o}{\eta} = M\eta + N(\eta) \tag{4}$$

with condition (3). A solution to (4) is $\eta = \eta(t)$ that is a stochastic K-process and each of the processes is considered to be equated, if almost surely each trajectory of one of the processes coincides with a process otherwise.

1. Analysis of Stochastic Mathematical Model

Consider a complete probability space $\Omega \equiv (\Omega, \mathrm{A}, P)$ and the set of real numbers $\mathbb{R}$ endowed with a Borel σ-algebra. The set of random variables (a measurable mapping $\xi : D \to \mathbb{R}$) with zero expectations (i.e. $\mathsf{E}\xi = 0$) and finite variance forms the Hilbert space L_2 (i.e. $\mathsf{D}\xi < +\infty$) with the inner product $(\xi_1, \xi_2) = \mathsf{E}\xi_1\xi_2$. Here E, D are the expectation and variance of the random variable, respectively. A mapping $\eta : \mathsf{I} \times \Omega \to \mathbb{R}$ of the form $\eta = \eta(t,\omega) = g(f(t),\omega)$ is called an (*one-dimensional*) *random process*, where $f : \mathsf{I} \to \mathsf{L}_2$ ($\mathsf{I} \subset \mathbb{R}$ is some set) and $g : \mathsf{L}_2 \times \Omega \to \mathbb{R}$. The set of continuous stochastic processes forms a Banach space $C(\mathsf{I}, \mathsf{L}_2)$.

Consider a real separable Hilbert space $(\mathrm{H}, <\cdot,\cdot>)$ identified with its conjugate space with the orthonormal basis $\{\varphi_k\}$. Let's write down $x = \sum\limits_{k=1}^{\infty} < x, \varphi_k > \varphi_k$ for each element $x \in \mathrm{H}$. Next, choose a monotonely decreasing numerical sequence $K = \{\mu_k\}$ such that $\sum\limits_{k=1}^{\infty} \mu_k^2 < +\infty$. Consider a sequence of random variables $\{\xi_k\} \subset \mathsf{L}_2$ such that $\sum\limits_{k=1}^{\infty} \mu_k^2 D\xi_k < +\infty$. Denote by $\mathrm{H}_K\mathsf{L}_2$ the Hilbert space of *random* K-*variables* of the form $\xi = \sum\limits_{k=1}^{\infty} \mu_k \xi_k \varphi_k$. Moreover, there is a random K-variable $\xi \in \mathrm{H}_K\mathsf{L}_2$. Note that dot prouct in $\mathrm{H}_K\mathsf{L}_2$ has form $(\xi^1, \xi^2) = \sum\limits_{k=1}^{\infty} \mu_k^2 \mathsf{E}\xi_k^1\xi_k^2$. Consider a sequence of random processes $\{\eta_k\} \subset C(\mathsf{I}, \mathsf{L}_2)$ and define the H-*valued continuous stochastic* K-*process*

$$\eta(t) = \sum_{k=1}^{\infty} \mu_k \eta_k(t) \varphi_k, \tag{5}$$

which is denoted by $C^1(\mathsf{I}; \mathrm{H}_K\mathsf{L}_2)$ and

$$\eta_0 = \sum_{k=1}^{\infty} \mu_k \eta_{0k} \varphi_k. \tag{6}$$

Note that the Nelson–Gliklikh derivatives of the random K-process

$$\overset{o}{\eta}(t) = \sum_{k=1}^{\infty} \mu_k \overset{o}{\eta}_k(t) \varphi_k \tag{7}$$

inclusively in the right-hand side, and all series converge uniformly in the norm $\mathrm{H}_K\mathsf{L}_2$ on any compact from I. Next, consider the space $C^1(\mathsf{I}; \mathrm{H}_K\mathsf{L}_2)$ of continuous stochastic K-processes whose trajectories are almost surely continuously differentiable by Nelson–Gliklikh.

Consider dual pairs of reflexive Banach spaces $(\mathsf{N}, \mathsf{N}^*)$ and $(\mathsf{B}, \mathsf{B}^*)$, where $\mathsf{N} = \overset{\circ}{W}{}_4^1(D)$, $\mathsf{B} = \overset{\circ}{W}{}_2^1(D)$, $\mathrm{H} = L_2(D)$ (note that B^* and N^* are dual spaces to B and N) are defined in the domain D such that the embeddings

$$\mathbf{N} \subset \mathbf{B} \subset \mathbf{H} \subset \mathbf{N}^* \subset \mathbf{B}^* \qquad (8)$$

are dense and continuous.

The solvability of the problem (1)–(3) in deterministic case is considered in [12]. Let us use similar reasoning for the stochastic case. Since model (1)–(3) refers to the deterministic case, we obtain the following splittings of the spaces:

$\mathbf{N} \supset \ker L \equiv coker L \subset \mathbf{N}^*$,	$\mathbf{N_K L_2} \supset [ker L]_\mathbf{K} \mathbf{L_2} \equiv [coker L]_\mathbf{K} \mathbf{L_2} \subset \mathbf{N^*_K L_2}$,
$\mathbf{N} = coimL \oplus \ker L$,	$\mathbf{N_K L_2} = [coimL]_\mathbf{K} \mathbf{L_2} \oplus [ker L]_\mathbf{K} \mathbf{L_2}$,
$\mathbf{N}^* = coker L \oplus imL$,	$\mathbf{N^*_K L_2} = [coker L]_\mathbf{K} \mathbf{L_2} \oplus [imL]_\mathbf{K} \mathbf{L_2}$,
$\mathbf{B} = ker L \oplus [\overline{coim} L \cap \mathbf{B}^*]$,	$\mathbf{B_K L_2} = [ker L]_\mathbf{K} \mathbf{L_2} \oplus [\overline{coim} L \cap \mathbf{B}^*]_\mathbf{K} \mathbf{L_2}$,
$\mathbf{B}^* = coker L \ \oplus [im L \cap \mathbf{B}^*]$,	$\mathbf{B^*_K L_2} = [coker L]_\mathbf{K} \mathbf{L_2} \oplus [im L \cap \mathbf{B}^*]_\mathbf{K} \mathbf{L_2}$.

In stochastic case, the operators L, M and N are defined as follows:

$$(L\eta, \zeta) = \int_D (\lambda \eta \zeta + \nabla \eta \cdot \nabla \zeta) ds \ \forall \ \eta, \zeta \in \mathbf{N_K L_2},$$

$$(M\eta, \zeta) = -a_1 \int_D \nabla \eta \cdot \nabla \zeta ds \ \forall \ \eta, \zeta \in \mathbf{N_K L_2},$$

$$(N(\eta), \zeta) = -a_2 \int_D |\nabla \eta|^2 \ \nabla \eta \cdot \nabla \zeta ds \ \forall \ \eta, \zeta \in \mathbf{B_K L_2},$$

where $(\cdot,\cdot)$ is the scalar product in $\mathrm{H}_K \mathbf{L_2}$. Note that $L: \mathbf{N} \to \mathbf{N}^*$ is a linear, continuous, self-adjoint, non-negatively defined and Fredholm operator in the deterministic case, then $L: \mathbf{N_K L_2} \to \mathbf{N^*_K L_2}$ it has the same properties. For all $a_1 \in \mathbf{R}$, $a_2 \in \mathbf{R}$ the operator $M: \mathbf{N_K L_2} \to \mathbf{N^*_K L_2}$ and $N: \mathbf{B_K L_2} \to \mathbf{B^*_K L_2}$ are dissipative. Similarly, we construct the spaces $\mathbf{N_K L_2}$. For an orthonormal basis, consider the sequence of eigenfunctions $\{\varphi_k\}$ and eigenvalues $\{\lambda_k\}$ for homogeneous Dirichlet problem for the Laplace operator $(-\Delta)$ in the domain D.

Let $\lambda \geq \lambda_1$

$$\ker L = \begin{cases} \{0\}, & \lambda > -\lambda_1; \\ span\{\varphi_1\}, & \lambda = -\lambda_1. \end{cases}$$

Then

$$[im \ \ L]_\mathbf{K} \mathbf{L_2} = \begin{cases} \mathbf{N^*_K L_2}, & \lambda > -\lambda_1; \\ \{\eta \in \mathbf{N^*_K L_2} : \ (\eta, \varphi_1) = 0\}, & \lambda = -\lambda_1, \end{cases}$$

$$[coim \ \ L]_\mathbf{K} \mathbf{L_2} = \begin{cases} \mathbf{N_K L_2}, & \lambda > -\lambda_1; \\ \{\eta \in \mathbf{N_K L_2} : \ (\eta, \varphi_1) = 0\}, & \lambda = -\lambda_1. \end{cases}$$

Suppose that $I \equiv (0, T)$. We use the space **H** in order to construct the spaces of K-“noises”, the spaces $C^1(\mathbf{I}; \mathrm{H}_K \mathbf{L_2})$ and $C^1(\mathbf{I}; \mathbf{N_K L_2})$, k $\in N$. Consider stochastic Sobolev type equation (4). A stochastic K-process $\eta \in C^1(\mathbf{I}; \mathbf{N_K L_2})$ is said to be a solution to equation (4), if almost surely all trajectories of η satisfy equation (4) for all $t \in I$. A solution $\eta = \eta(t)$ to equation (4) that satisfies the initial value condition

$$\lim_{t \to 0+} L(\eta(t) - \eta_0) = 0 \qquad (9)$$

is called a solution to Showalter–Sidorov problem (4), (9) for some random K-variable $\eta_0 \in \mathbf{N_K L_2}$. Fix $\omega \in \Omega$, since the solution of the problem is considered trajectory.

Theorem Let $\lambda \geq -\lambda_1$, $a_1, a_2 \in \mathbf{R}$, then for any $\eta_0 \in \mathbf{N_K L_2}$, there exists a unique solution $\eta \in C^1(\mathbf{I}; \mathbf{N_K L_2})$ to problem (1)–(3).

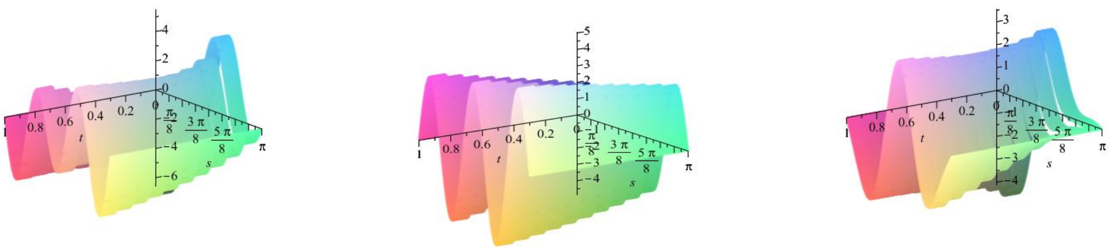
Proof. Since $\omega \in \Omega$ is fixed, then the proof of the theorem is equivalent to those in the deterministic case [12]. □

2. Process Information

In [13] an algorithm was proposed for numerical study of Sobolev type stochastic models with Showalter–Sidorov conditions. A feature of the study of models with random influence is that it is necessary to carry out m computational experiments. Each of them uses a normally distributed generator of random variables with specified parameters of mathematical expectation and variance. As a result, we get several implementations of the solution. According to the results of the experiment, the selective mathematical expectation $\mathbf{E}(\eta(s,t))$, for any value of t, is equal to the mathematical expectation of the corresponding part, i.e. the mean trajectory obtained as a result of processing m experience. In addition to showing the performance of the experiments, it is also necessary to check if they are within the confidence interval. The width of the confidence interval depends on the size of the standard error, which, in turn, depends on the sample size and, when considering a numerical variable from the variability of the data, give wider confidence intervals than studies of a large data set of few data. If the realizations lie within the confidence interval, then the results are consistent with this likely value. The probability that the confidence interval contains the realizations of the experiment is called the confidence probability (usually 0,95 or 0,99).

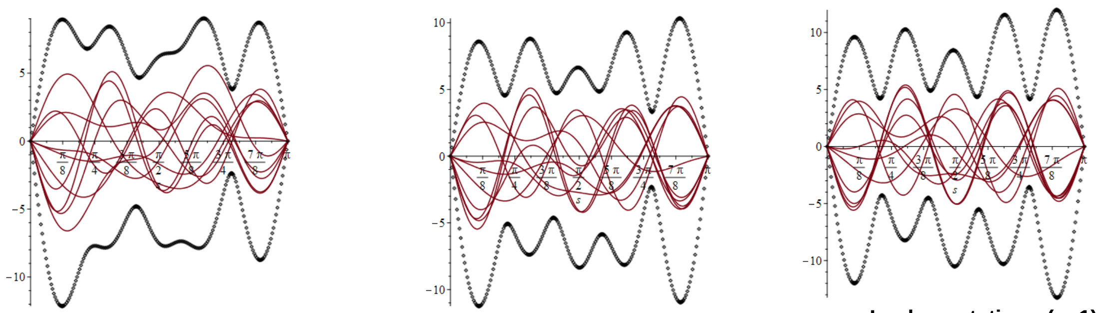
Consider problem (1)–(3) and represent the solution: $\eta_N(s,t)=\sum_{k=1}^{N}\mu_k\eta_k(t)\varphi_k(s)$. Initial random influence has form $\eta_{0N}=\sum_{k=1}^{N}\mu_k\eta_{0k}\varphi_k$. Let's set the initial data: $\lambda=1, a_1=1, a_2=1, \mu_k=1\backslash k^2$, the parameter T = 1 of the time interval [0, T], Galerkin approximation N = 5, the parameter of the random effect is the mathematical expectation of 0 and the standard deviation of 2, the domain $D=(0;\pi)$. For each experiment, initial random influence η_{0k} is randomly generated.

In Fig. 1, the graphs represent the function $\eta(s,t)$, i = 1, 5, 10. On Fig. 2 shows the execution of the estimate, the lines represent the graphs of the functions $\eta(s,t)$, i = 1, . . . , 10, at a fixed point in time; dotted lines show the boundaries of the confidence interval obtained numerically.



Experiment no. 1 **Experiment no. 5** **Experiment no. 10**

Fig. 1. Graphs of the function *η*(*s*, *t*), *i* = 1, 5, 10



Implementation *η*(*s*; 0) and confidence interval **Implementation *η*(*s*; 0,5) and confidence interval** **Implementation *η*(*s*; 1) and confidence interval**

Fig. 2. Implementation *η*(*s*; *t*) at a fixed point in time and confidence interval

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Received April 14, 2023

Information about the authors

Perevozchikova Ksenia Vladimirovna is Senior Lecturer, Department of Mathematical Physics Equations, South Ural State University, Chelyabinsk, Russian Federation, email: perevozchikovakv@susu.ru

Bulletin of the South Ural State University Series "Mathematics. Mechanics. Physics" 2023, vol. 15, no. 2, pp. 14–20

УДК 517.9 **DOI: 10.14529/mmph230202**

АНАЛИЗ И ОБРАБОТКА ИНФОРМАЦИИ ДЛЯ ОДНОЙ СТОХАСТИЧЕСКОЙ СИСТЕМЫ СОБОЛЕВСКОГО ТИПА

К.В. Перевозчикова
Южно-Уральский государственный университет, г. Челябинск, Российская Федерация
E-mail: perevozchikovakv@susu.ru

Аннотация. Статья посвящена анализу и обработке информации для стохастической модели, основанной на уравнении распределения потенциалов в кристаллическом полупроводнике с производной Нельсона–Гликлиха и начальным условием Шоуолтера–Сидорова. Под полупроводником мы будем понимать вещества, обладающие конечной электропроводностью, быстро возрастающей с ростом температуры. Предполагается, что на экспериментальные начальные данные возможно влияние случайных помех, которые приводят к исследованию стохастической модели. В работе приведен анализ полученной стохастической модели распределения потенциалов в кристаллическом полупроводнике. Найдены условия, при которых существует потраекторное существование решений исследуемой модели с начальным условием Шоуолтера–Сидорова. На базе теоретических результатов разработан алгоритм численного анализа системы и представлена его реализация в виде вычислительного эксперимента, который необходим для дальнейшей обработки информации.

Ключевые слова: стохастическая модель распределения потенциалов в кристаллическом полупроводнике; анализ и обработка информации; производная Нельсона–Гликлиха; уравнения соболевского типа.

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Поступила в редакцию 14 апреля 2023 г.

Сведения об авторах

Перевозчикова Ксения Владимировна – старший преподаватель, кафедра уравнений математической физики, Южно-Уральский государственный университет, г. Челябинск, Российская Федерация, e-mail: perevozchikovakv@susu.ru