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ANALYSIS OF THE STOCHASTIC WENTZELL SYSTEM OF FLUID FILTRATION EQUATIONS IN A CIRCLE AND ON ITS BOUNDARY

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Abstract. Wentzell boundary condition problems for linear elliptic equations of second order have been studied by various methods. Over time, the condition has come to be understood as a description of a process occurring on the boundary of a domain and affected by processes inside the domain. Since Wentzell boundary conditions in the mathematical literature have been considered from two points of view (in the classical and neoclassical cases), the aim of this paper is to analyse the stochastic Wentzell system of filtration equations in a circle and on its boundary in the space of differentiable K-"noise". In particular, we prove the existence and uniqueness of the solution that determines quantitative predictions of changes in the geochemical regime of groundwater in the case of non-pressure filtration at the boundary of two media (in the region and on its boundary).
Keywords: Wentzell system; filtration equation; Nelson–Glicklich derivative; Wentzell boundary conditions.

Introduction

Liquid filtration as well as its flow, diffusion, falling, etc. is one of the moisture transfer processes. The study of these processes begins with the study of their mathematical models. Let us consider one of the mathematical models of filtration. Let $\Omega \in \mathbb{R}^n$, $n \geq 2$ be a connected bounded region with boundary Γ of class C^∞. The system of Barenblatt–Zheltov–Kochina equations [1], modelling the process of fluid filtration is defined on the compact $\Omega \cup \Gamma$

$$(\lambda - \Delta)u_t = \alpha\Delta u + \beta u, u = u(t,x), (x,t) \in \mathbb{R} \times \Omega, \tag{1}$$

$$(\lambda - \Delta)v_t = \gamma\Delta v + \frac{\partial u}{\partial \nu} + \delta v, v = v(t,x), (x,t) \in \mathbb{R} \times \Gamma, \tag{1}$$

$$\operatorname{tr} u = v \ on \ \mathbb{R} \times \Gamma. \tag{3}$$

Here the symbol Δ in (1) denotes the Laplace operator in the region Ω, and in (2) the same symbol denotes the Laplace – Beltrami operator on the smooth Riemannian manifold Γ. The symbol $\nu = \nu(t,x), (t,x) \in \mathbb{R} \times \Gamma$, denotes the normal to $\mathbb{R} \times \Omega$ external to $\mathbb{R} \times \Gamma$. The parameters $\lambda, \alpha, \beta, \gamma, \delta \in \mathbb{R}$ characterise the medium.

Previously [2], following the tradition of [3–7], we called the condition of the form (2), in which the order of derivatives on spatial variables is not lower than the order on the same ones in (1), the Wentzell boundary condition. However, intending in the future to consider different cases of Ω and Γ (for instance, Ω is a bounded connected Riemannian manifold with edge Γ) we consider it necessary to call (1), (2) a system of equations, albeit defined on sets of different geometric dimension. This is supported by the fact that equations (1), (2) describe the same physical process of fluid filtration. The term "boundary conditions" should be reserved for the equations defined on the boundary (edge) of a region (manifold) and having a lower order of derivatives on spatial variables (see the classical treatise [8]). The name of the system of Wentzell equations emphasises the merits of the discoverer [9] of a new section of mathematical physics.

We will study the solvability of the system (1), (2) in the simplest case: $\Omega = \{(r,\theta) : r \in [0,R), \theta \in [0,2\pi)\}$ is a circle, and $\Gamma = \{\theta : \theta \in [0,2\pi)\}$ is a circumference. In this case (1), (2) is transformed to the form

$$(\lambda - \Delta_{r,\theta})u_t = \alpha\Delta_{r,\theta}u + \beta u, u = u(t,r,\theta), (t,r,\theta) \in \mathbb{R} \times \Omega, \tag{4}$$

$$(\lambda - \Delta_\theta)v_t = \gamma\Delta_\theta v + \partial_R u + \delta v, v = v(t,\theta), (t,\theta) \in \mathbb{R} \times \Gamma, \tag{5}$$

in which

$$\Delta_{r,\theta} = (r-R)\frac{\partial}{\partial r}\left((R-r)\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial\theta^2}, \Delta_\theta = \frac{\partial^2}{\partial\theta^2}, \partial_R = \frac{\partial}{\partial r}\bigg|_{r=R}.$$

To the system (4), (5) we add the matching condition (3) and provide it with initial conditions

$$u(0,r,\theta) = u_0(r,\theta), v(0,\theta) = v_0(\theta). \tag{6}$$

The solution of the problem (3)–(6) we denote as a deterministic solution. It should be noted that transforming the operator (4) to Cartesian coordinates we obtain

$$\Delta_{x,y} = \left(x^2+2y^2\right)\frac{\partial^2}{\partial x^2} + \left(y^2+2x^2\right)\frac{\partial^2}{\partial y^2}.$$

We transfer the consideration of the standard situation to our future research.

Resorting to the stochastic interpretation of partial derivative equations, in this paper we are going to touch upon the studies of nondeterministic problems in the interpretation necessary for us, the distinguishing feature of which is a different notion of "white noise" in the sense of the Nelson–Glicklich derivative of the Wiener process. The term Nelson–Glicklich derivative was originally introduced in the monograph [10], where the first derivative of a random process was found. This paradigm not only justified the consistency with the Einstein–Smoluchowski theory, which allows us to understand by Brownian motion the sought stochastic process, and by the derivative from this process – "white noise", but also prompted the emergence of a new direction of study of stochastic equations of the Sobolev type. This is reflected in the studies of: dichotomies of a stochastic equation defined on a manifold [11]; application of the phase space method in the case of (L, p)-bounded operator M in [12]; stochastic equations of Sobolev type of high order in [12].

The paper consists of two parts. In the first part the existence and uniqueness of the system of Wentzell equations in a circle and on its boundary are considered. The second part contains abstract reasoning consisting in the construction of space and proof of existence and uniqueness of the stochastic system of Wentzell equations in a circle and on its boundary.

Deterministic case

Since it is not difficult to notice,

$$u = \sum_{k=2}^{\infty}\exp\left(t\frac{\beta-\alpha k^2}{\lambda+k^2}\right)\frac{(R-r)^k}{2R^k}\left(a_k\cos k\theta + b_k\sin\kappa\theta\right) + \sum_{k=1}^{\infty}\exp\left(t\frac{\beta-\alpha k^2}{\lambda+k^2}\right)\left(c_k\cos k\theta + d_k\sin\kappa\theta\right), \tag{7}$$

in which

$$a_k = \int_0^R\int_0^{2\pi} u_0(r,\theta)\frac{(R-r)^k}{2R^k}\cos k\theta d\theta r dr,$$

$$b_k = \int_0^R\int_0^{2\pi} u_0(r,\theta)\frac{(R-r)^k}{2R^k}\sin k\theta d\theta r dr,$$

$$c_k = \int_0^{2\pi} v_0(\theta)\cos k\theta d\theta, d_k = \int_0^{2\pi} v_0(\theta)\sin k\theta d\theta,$$

is the formal solution of equation (4). If the series in (7) converge uniformly, then we have a solution of the problem (4), (6), with $\partial_R u = 0$. Taking this into account, we can construct the solution of problem (5), (6)

$$v = \sum_{k=2}^{\infty}\exp\left(t\frac{\delta-\gamma k^2}{\lambda+k^2}\right)\left(c_k\cos k\theta + d_k\sin\kappa\theta\right), \tag{8}$$

and in the case $\alpha=\gamma, \beta=\delta$ the solutions of the problem (4)–(6) will satisfy the matching condition (3).

Further, the closure of the linear

$$\operatorname{span}\left\{\left(2R^k\right)^{-1}(R-r)^k\cos k\theta, \left(2R^k\right)^{-1}(R-r)^k\sin k\theta : k\in N\setminus\{1\}, r\in(0,R), \theta\in[0,2\pi)\right\}$$

by the norm generated by the scalar product of

$$(\phi,\psi)=\int\limits_0^{2\pi}\int\limits_0^R\varphi(r,\theta)\psi(r,\theta)rdrd\theta,$$

we denote by the symbol $A(\Omega)$. Closure of the linear $\operatorname{span}\{\cos k\theta,\sin k\theta : k\in N,\theta\in[0,2\pi)\}$ by the norm generated by the scalar product of

$$(\varphi,\psi)=\int\limits_0^{2\pi}\varphi(\theta)\psi(\theta)d\theta,$$

we denote by the symbol $A(\Gamma)$.

Theorem 2.1 *For any $u_0\in A(\Omega)$ and $v_0\in A(\Gamma)$ such that (3) is satisfied, and any $\alpha,\beta,\gamma,\delta,\lambda\in R$, such that $\alpha=\gamma,\beta=\delta$, and $\lambda\neq k^2$, in which $k\in N$, there exists a singular solution $(u,v)\in C^\infty(R;A(\Omega)+A(\Gamma))$ of problem (3)–(5).*

The existence of the solution is proved by formulas (7)–(8), the proof of the singularity of the solution is trivial.

Stochastic case

Let $\Omega\equiv(\Omega,A,P)$ be a complete probability space with probability measure P associated to the σ-algebra A of subsets of the set Ω, and let R be the set of real numbers endowed with a Borel σ-algebra. A measurable mapping $\xi:\Omega\to R$ is called a *random variable*. The set of random variables whose expectation E is zero and variance D is finite forms a Hilbert space $L_2=\{\xi : E\xi=0, D\xi<+\infty\}$ with scalar product $(\xi_1,\xi_2)=E\xi_1\xi_2$ and a norm $\|\xi\|^2_{L_2}=D\xi$.

Let us take the set $T\subset R$ and consider two mappings: $f:T\to L_2$, which maps a random variable $\xi\in L_2$ to $t\in T$, and $g:L_2\times\Omega\to\mathbb{R}$, which maps a point $\xi(\omega)\in\mathbb{R}$ to each pair (ξ,ω). The mapping $\eta:T\times\Omega\to\mathbb{R}$, which has the form $\eta=\eta(t,\omega)=g(f(t),\omega)$, we will call *a (one-dimensional) stochastic process*. Considering $T\subset\mathbb{R}$ an interval, we call a stochastic process $\eta=\eta(t)$, $t\in T$, *continuous if a.b. (almost probably) all its trajectories are continuous (i.e., if a.a. (almost all)* $\omega\in A$ trajectories $\eta(\cdot,\omega)$ are continuous functions). The set of continuous stochastic processes forms a Banach space, which we denote by the symbol $C(T;L_2)$ with a norm

$$\|\eta\|_{CL_2}=\sup_{t\in I}(\mathbf{D}\eta(t,\omega))^{1/2}.$$

Let us consider the properties of the Nelson–Glicklich derivative $\overset{\circ}{\eta}$ of a stochastic process η at the point $t\in T$ (for a detailed description, see for example in monograph [9]). If the Nelson–Glicklich derivatives $\overset{\circ}{\eta}(t,\cdot)$ of a stochastic process $\eta(t,\cdot)$ exist at all (or almost all) points of an interval T, then we refer to the existence of a Nelson–Glicklich derivative $\overset{\circ}{\eta}(t,\cdot)$ on T (almost probably on T). The set of stochastic processes, whose trajectories are Nelson–Glicklich differentiable on I up to order $l\in\{0\}\cup N$ inclusively form a Banach space $C^l(T;L_2)$, $l\in N$ with a norm

$$\|\eta\|_{C^lL_2}=\sup_{t\in I}\left(\sum_{k=0}^{l}\mathbf{D}\overset{\circ}{\eta}^{(k)}(t,\omega)\right)^{1/2}.$$

Here we will consider the zero-order Nelson–Glicklich derivative as the initial random process, i.e. $\overset{\circ}{\eta}^{(0)}\equiv\eta$. Let us also note that the spaces $C^l(T;L_2)$, $l\in\{0\}\cup N$, will be called “noise” spaces for simplicity. (see, example, [3–7]).

Let us proceed to the construction of the space of *random* K *-values.* We consider H a real separable Hilbert space with orthonormalised basis $\{\varphi_k\}$, a monotone sequence $K=\{\lambda_k\}\subset\mathbb{R}_+$ such that $\sum_{k=1}^{\infty}\lambda_k^2<+\infty$, and a sequence $\{\xi_k\}=\xi_k(\omega)\subset L_2$ of random variables such that $\|\xi_k\|\le C$, for some constant $C\in\mathbb{R}_+$ and for all $k\in N$. Let us construct a H *-valued* K *-value*

$$\xi(\omega)=\sum_{k=1}^{\infty}\lambda_k\xi_k(\omega)\varphi_k.$$

Completion of the linear envelope of the set $\{\lambda_k\xi_k\varphi_k\}$ by the norm

$$\|\eta\|_{H_K L_2}=\left(\sum_{k=1}^{\infty}\lambda_k^2 D\xi_k\right)^{1/2}$$

is called the space of *(* H *-valued) random* K *-values and is denoted by the symbol* $H_K L_2$. As we can clearly observe, the space $H_K L_2$ is Hilbertian, and the above constructed random K-value $\xi=\xi(\omega)\in H_K L_2$ Similarly, the Banach space of (H *-valued*) K *-"noises"* $C^l(T;H_K L_2)$, $l\in\{0\}\cup N$, let us define it as an enlargement of the linear envelope of the set $\{\lambda_k\eta_k\varphi_k\}$ by the norm

$$\|\eta\|_{C^l H_K L_2}=\sup_{t\in I}\left(\sum_{k=1}^{\infty}\lambda_k^2\sum_{m=1}^{l}D\overset{\circ}{\eta}{}_k^{(m)}\right)^{1/2},$$

where the sequence of "noises" $\{\eta_k\}\subset C^l(T;L_2)$, $l\in\{0\}\cup N.$ As we can clearly see, the vector

$$\eta(t,\omega)=\sum_{k=1}^{\infty}\lambda_k\eta_k(t,\omega)\varphi_k$$

lies in the space $C^l(T;H_K L_2)$, if the sequence of vectors $\{\eta_k\}\subset C^l(T;L_2)$ and all their Nelson–Glicklich derivatives up to and including order $l\in\{0\}\cup N$ are uniformly bounded by the norm $\|\cdot\|_{C^l L_2}$.

Now let A (F) be a real separable Hilbert space with orthonormalised basis $\{\varphi_k\}$ ($\{\psi_k\}$). Let us introduce a monotone sequence $K=\{\lambda_k\}\subset\{0\}\cup R$ such that $\sum_{k=1}^{\infty}\lambda_k^2<+\infty$. By the symbol $U_K L_2$ ($F_K L_2$**)** we denote the Hilbert space which is a replenishment of the linear envelope of random K *-values*

$$\xi=\sum_{k=1}^{\infty}\lambda_k\xi_k\varphi_k,\xi_k\in L_2,\left(\zeta=\sum_{k=1}^{\infty}\mu_k\zeta_k\psi_k,\zeta_k\in L_2\right)$$

by the norm

$$\|\eta\|_U^2=\sum_{k=1}^{\infty}\lambda_k^2 D\xi_k\left(\|\omega\|_F^2=\sum_{k=1}^{\infty}\mu_k^2 D\zeta_k\right).$$

It should be noted that in different spaces ($U_K L_2$ и $F_K L_2$) the sequence K can be different ($K=\{\lambda_k\}$ in $U_K L_2$ and $K=\{\mu_k\}$ in $F_K L_2$), but all sequences marked by K must be monotone and summable with square. All results will, in general, be true for different sequences $\{\lambda_k\}$ и $\{\mu_k\}$, but for simplicity's sake we will restrict ourselves to the case $\lambda_k=\mu_k$.

Let $A:A\to F$ be a linear operator. By the formula

$$A\xi=\sum_{k=1}^{\infty}\lambda_k\xi_k A\varphi_k \tag{9}$$

we define a linear operator $A:U_K L_2\to F_K L_2$, and if the series in the right-hand side of (9) converges (in the metric $F_K L_2$) then $\xi\in\operatorname{dom}A$, and if it diverges, then $\xi\notin\operatorname{dom}A$. Traditionally, the spaces of linear continuous operators $L(U_K L_2;F_K L_2)$ and linear closed densely defined operators are defined. The following is valid

Lemma 3.1 (i) *An operator* $A \in L(A;F)$ *exactly and only if* $A \in L(U_K L_2; F_K L_2)$

As it can be easily observed,

$$\|A\xi\|_F \le \sum_{k=1}^{\infty} \lambda_k^2 D\xi_k \|A\varphi_k\|_F^2 \le \text{const} \sum_{k=1}^{\infty} \lambda_k^2 D\xi_k = \text{const}\|\xi\|_U$$

(ii) *the operator* $A \in Cl(A;F)$ *exactly and only if* $A \in Cl(U_K L_2; F_K L_2)$.

Lemma 3.2 *The operator* $M \in Cl(A;F)$ *is* p*-limited with respect to the operator* $L \in L(A;F)$ *exactly and only if* $M \in Cl(U_K L_2; F_K L_2)$ *is* p*-limited with respect to the operator* $L \in L(U_K L_2; F_K L_2)$*. Moreover, the relative spectrum is the same in both cases.*

For simplicity sake, let $A = \{u \in W_2^2(\Omega) + W_2^2(\Gamma) : \partial_R u = 0\}$, $F = L_2(\Omega) + L_2(\Gamma)$. Next, following the prescriptions above, we construct the spaces of random K-values $U_K L_2$ и $F_K L_2$. Random K-value $\xi \in U_K L_2$ has a form

$$\xi = \sum_{k=1}^{\infty} \lambda_k \xi_k \varphi_k.$$

In which $\{\varphi_k\}$ is the family of eigenfunctions of the Laplace operator $\Delta_{r,\theta} : A \to F$ orthonormalised in the sense of the scalar product $(\cdot,\cdot)$ from $L_2(\Omega)$. Consider the linear stochastic Wentzel system of the fluid filtration equation in a circle and at its boundary. In this case (1), (2) is transformed to the form

$$(\lambda - \Delta_{r,\theta})\eta_t = \alpha\Delta_{r,\theta}\eta + \beta\eta, \eta \in C^\infty \tag{10}$$

$$(\lambda - \Delta_\theta)\eta_t = \gamma\Delta_\theta\eta + \partial_R\eta + \delta\eta, \eta \in C^\infty \tag{11}$$

in which

$$\Delta_{r,\theta} = (r-R)\frac{\partial}{\partial r}\left((R-r)\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial\theta^2}, \Delta_\theta = \frac{\partial^2}{\partial\theta^2}, \partial_R = \left.\frac{\partial}{\partial r}\right|_{r=R}.$$

To the system (10), (11) we add the initial condition

$$\eta(0) = \eta_0. \tag{12}$$

The solution of the problem (11)–(12) we will call a stochastic solution.

Theorem 3.1 *For any* $\eta_0 \in U_K L_2(\Omega)$ *and any* $\alpha, \beta, \gamma, \delta, \lambda \in R$*, such that* $\alpha = \gamma, \beta = \delta$*, a* $\lambda \ne k^2$*, in which* $k \in N$*, there exists a single solution* $\eta \in C^\infty(\mathbb{R}_+; U_k L_2)$ *of problem (10)–(12).*

The existence and singleness of the solution are proved by analogy with the deterministic case by virtue of Lemmas 3.1 and 3.2.

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АНАЛИЗ СТОХАСТИЧЕСКОЙ СИСТЕМЫ ВЕНТЦЕЛЯ УРАВНЕНИЙ ФИЛЬТРАЦИИ ЖИДКОСТИ В КРУГЕ И НА ЕГО ГРАНИЦЕ

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Аннотация. Задачи с граничным условием Вентцеля для линейных эллиптических уравнений второго порядка изучались различными методами. Со временем условие стало пониматься как описание процесса, происходящего на границе области и на который влияют процессы внутри области. Поскольку в математической литературе граничные условия Вентцеля рассматривались с двух точек зрения (в классическом и неоклассическом случаях), целью данной работы является анализ стохастической системы Вентцеля уравнений фильтрации в круге и на его границе в пространстве дифференцируемых K-«шумов». В частности, доказано существование и единственность решения, которое определяет количественные прогнозные изменения геохимического режима грунтовых вод при безнапорной фильтрации, протекающей на границе двух сред (в области и на ее границе).

Ключевые слова: система Вентцеля; уравнение фильтрации; производная Нельсона–Гликлиха; краевые условия Вентцеля.

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