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NUMERICAL SIMULATION OF THE DEFLECTION OF A RECTANGULAR PLATE ON AN ELASTIC BASE WITH ITS RIGID EDGE FIXATION

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Abstract. The paper presents an algorithm for numerically modeling the deflection of a rectangular plate on an elastic foundation with rigid fixation at its edges. The proposed algorithm is asymptotically optimal in terms of computational effort and is based on the iterative expansion method and assumes the use of marching methods. The asymptotic optimality of the algorithm has been experimentally confirmed using computer simulations.

Keywords: numerical modeling; deflection of a rectangular plate.

Introduction

Numerical modeling of the deflection of a rectangular plate on an elastic foundation with its rigid fixation at the edges presents practical problems [1]. In the best cases, logarithmically optimal algorithms for solving such a problem are known, although the development of asymptotically optimal algorithms is theoretically possible [2, 3]. In the present paper, an algorithm of this type is proposed based on the use of the iterative extension method [4] and the proposed application of well-known marching methods [5–8]. Computer calculations have been performed, experimentally confirming the asymptotic optimality of the developed algorithm.

Practical task

In real three-dimensional space it is considered under transverse load, under pressure P_1 rectangular plate with the stiffness coefficient of the elastic base $K_1 \geq 0,$ with rigidly fixed edges, length $l_1,$ width $b_1,$ thickness $H,$ when $0 < H << b_1 \leq l_1$ made of material with Young's modulus $E_1 > 0$ and Poisson's ratio $\sigma_1 \in [0;1).$ Find the middle surface of the plate, equidistant, for example, from the lower and upper surfaces of the plate, when the plate is located horizontally.

Regions

It is assumed that the first region is given and the second region is selected that does not intersect with the first region. $\Omega_\omega \subset R^2,\ \omega \in \{1, \mathrm{II}\},$ and the union of the closures of these regions is the closure of a square region $\Pi.$ The boundaries of the regions consist of open arcs:

$$\partial\Pi = \bar{s},\ s = \Gamma_1 \cup \Gamma_2,\ \Gamma_1 \cap \Gamma_2 = \varnothing,$$

$$\partial\Omega_1 = \bar{s}_1,\ \partial\Omega_{\mathrm{II}} = \bar{s}_{\mathrm{II}},\ s_1 = \Gamma_{1,0},\ s_{\mathrm{II}} = \Gamma_{\mathrm{II},1} \cup \Gamma_{\mathrm{II},2} \cup \Gamma_{\mathrm{II},3},\ \Gamma_{1,0} \cap \Gamma_{\mathrm{II},3} \neq \varnothing.$$

In this case, the intersection of the boundaries of the first and second regions is as follows

$$\partial\Omega_1 \cap \partial\Omega_{\mathrm{II}} = \bar{S},\ S = \Gamma_{1,0} \cap \Gamma_{\mathrm{II},3} \neq \varnothing.$$

We will consider such areas

$$\Pi = (-l_2; l_3) \times (-b_2; b_3).\ \ \Omega_1 = (0; l_1) \times (0; b_1),\ 0 < b_1 \leq l_1,$$

$$\Omega_{\mathrm{II}} = (-l_2; l_3) \times (-b_2; b_3) \setminus [0; l_1] \times [0; b_1],\ 0 < l_2,\ l_1 < l_3,\ 0 < b_2,\ b_1 < b_3,$$

The first region is an open rectangle, and the second region is an open and larger rectangle with the closure of the first region punctured. The boundaries of the regions contain the following parts:

$$\Gamma_1 = \{l_3\} \times (-b_2; b_3) \cup (-l_2; l_3) \times \{b_3\},\ \Gamma_2 = \{-l_2\} \times (-b_2; b_3) \cup (-l_2; l_3) \times \{-b_2\}.$$

$$\Gamma_{1,0} = \{0,\ l_1\} \times (0; b_1) \cup (0; l_1) \times \{0,\ b_1\},$$

$$\Gamma_{\mathrm{II},1} = \{l_3\} \times (-b_2; b_3) \cup (-l_2; l_3) \times \{b_3\},\ \Gamma_{\mathrm{II},2} = \{-l_2\} \times (-b_2; b_3) \cup (-l_2; l_3) \times \{-b_2\},$$

$$\Gamma_{\mathrm{II},3} = \{0,\ l_1\} \times (0; b_1) \cup (0; l_1) \times \{0,\ b_1\},$$

Solved and fictitious problems

In the first region, i. e. when $\omega = 1$ the problem to be solved is set. In the second area, i. e. when $\omega = \text{II}$ then a fictitious homogeneous problem is introduced:

$$D_1\Delta^2\breve{u}_1 + K_1\breve{u}_1 = P_1 \text{ in } (0;l_1)\times(0;b_1),\ \breve{u}_1 = \frac{\partial\breve{u}_1}{\partial\vec{n}} = 0 \text{ on } \{0,\ l_1\}\times(0;b_1)\cup(0;l_1)\times\{0,\ b_1\}, \qquad (1)$$

where $D_1 = E_1H^3/(12(1-\sigma_1^2)) > 0$ is cylindrical rigidity of the plate. If we assume that $f_1 = P_1/D_1$, $a_1 = K_1/D_1 \ge 0$, then the problem is solvable

$$\Delta^2\breve{u}_1 + a_1\breve{u}_1 = f_1 \text{ in } (0;l_1)\times(0;b_1),\ \breve{u}_1 = \frac{\partial\breve{u}_1}{\partial\vec{n}} = 0 \text{ on } \{0,\ l_1\}\times(0;b_1)\cup(0;l_1)\times\{0,\ b_1\}. \qquad (2)$$

Here $\frac{\partial\breve{u}_1}{\partial\vec{n}}$ – derivative with respect to the outward normal of a function $\breve{u}_1$, and $\breve{u}_1$ function of displacement of points of the midplane of the plate. The plate is considered under transverse pressure P_1, determining the load f_1 and is located on an elastic foundation, with rigid fixation at the boundary. The solved and fictitious problems are considered in variational form:

$$\breve{u}_\omega \in \breve{H}_\omega:\ \Lambda_\omega(\breve{u}_\omega,\breve{v}_\omega) = F_\omega(\breve{v}_\omega)\ \forall\breve{v}_\omega \in \breve{H}_\omega,\ \omega\in\{1,\ \text{II}\} \qquad (3)$$

spaces of Sobolev functions.

$$\breve{H}_1 = \breve{H}_1(\Omega_1) = \left\{\breve{v}_1 \in W_2^2(\Omega_1) : \breve{v}_1\big|_{\Gamma_{1,0}} = \frac{\partial\breve{v}_1}{\partial\vec{n}}\Big|_{\Gamma_{1,0}} = 0\right\},$$

$$\breve{H}_{\text{II}} = \breve{H}_{\text{II}}(\Omega_{\text{II}}) = \left\{\breve{v}_{\text{II}} \in W_2^2(\Omega_{\text{II}}) : \breve{v}_{\text{II}}\big|_{\Gamma_{\text{II},1}} = 0,\ \frac{\partial\breve{v}_{\text{II}}}{\partial\vec{n}}\Big|_{\Gamma_{\text{II},2}} = 0\right\},$$

where the right-hand sides of the problems are functionals

$$F_\omega(\breve{v}_\omega) = (\breve{f}_\omega,\breve{v}_\omega),\ (\breve{f}_\omega,\breve{v}_\omega) = \int_{\Omega_\omega}\breve{f}_\omega\breve{v}_\omega d\Omega_\omega,\ \breve{f}_{\text{II}} = 0,$$

and the left parts of the problems are bilinear forms

$$\Lambda_\omega(\breve{u}_\omega,\breve{v}_\omega) = \int_{\Omega_\omega}(\breve{u}_{\omega xx}\breve{v}_{\omega xx} + 2\breve{u}_{\omega xy}\breve{v}_{\omega xy} + \breve{u}_{\omega yy}\breve{v}_{\omega yy} + a_\omega\breve{u}_\omega\breve{v}_\omega)d\Omega_\omega,\ a_{\text{II}} > a_1 \ge 0.$$

Each of these problems has a unique solution [9]. The solution to the fictitious problem is zero.

To jointly denote the solution, the right side of the original problem $\breve{u}_1$, f_1 and solutions, the right side of the fictitious problem $\breve{u}_{\text{II}}$, $f_{\text{II}} = 0$ the following designations can be used accordingly $\breve{u}$, f, omitting the index ω. So often, for convenience, a function and its continuation are designated the same way.

Difference approximation

In the previously entered rectangular area

$$\Pi = (-l_2;l_3)\times(-b_2;b_3),\ \ \Gamma_1 = \{l_3\}\times(-b_2;b_3)\cup(-l_2;l_3)\times\{b_3\},\ \Gamma_2 = \{-l_2\}\times(-b_2;b_3)\cup(-l_2;l_3)\times\{-b_2\}$$

and on the encircling strip we will introduce a grid with nodes:

$$(x_i;y_j) = \big((i-m-0{,}5)h+\delta_1;(j-n-0{,}5)h\big),\ h = b_1/n,\ m = [l_1/h],\ \delta_1 = (l_1-mh)/2,\ h << b_1 \le l_1,$$
$$i = 0,...,3m,\ j = 0,...,3n,\ 3\le m,n,\ m,n\in\mathrm{N}.$$

We believe that

$$-l_2 = (x_0+x_1)/2 = -mh+\delta_1 = -l_1+3\delta_1,\ 0\le\delta_1 < h/2,\ l_2 = l_1-3\delta_1,$$
$$l_3 = x_{3m} = (2m-0{,}5)h+\delta_1 = 2(l_1-2\delta_1)-h/2+\delta_1 = 2l_1-3\delta_1-h/2,\ l_3 = 2l_1-3\delta_1-h/2,$$
$$-b_2 = (y_0+y_1)/2 = nh = -b_1,\ b_2 = b_1,\ \ b_3 = y_{3n} = (2n-0{,}5)h = 2b_1-h/2,\ b_3 = 2b_1-h/2.$$

We will consider arrays with values of grid functions at the nodes of the previously introduced grid.

$$v_{i,j} = v(x_i;y_j)\in R,\ i = 0,1...,3m,\ j = 0,1...,3n.$$

Note that with discrete approximation $-\Delta\breve{u}$ on Π without taking into account the boundary conditions, it is replaced by systems of differences using the approximation method by parts [9]:

$$\frac{4u_{i,j}-u_{i-1,j}-u_{i+1j}-u_{i,j-1}-u_{i,j+1}}{h^2},\ 1\le i\le 3m-1,\ 1\le j\le 3n-1.$$

After multiplying by h^2 in the difference scheme approximating $-\Delta\breve{u}$, the result is the so-called cross pattern with coefficients:

$$\begin{array}{cccc} & j-1 & j & j+1 \\ i-1 & & -1 & \\ i & -1 & 4 & -1 \\ i+1 & & -1 & \end{array}.$$

With discrete approximation already $\Delta^2\breve{u}$ on Π without taking into account the boundary conditions, if in differences we multiply by h^4, It turns out that the so-called big cross pattern is used with coefficients:

$$\begin{array}{cccccc} & j-2 & j-1 & j & j+1 & j+2 \\ i-2 & & & 1 & & \\ i-1 & & 2 & -8 & 2 & \\ i & 1 & -8 & 20 & -8 & 1 \\ i+1 & & 2 & -8 & 2 & \\ i+2 & & & 1 & & \end{array}.$$

We do not use, for example, the usual numbering of vector components $v_{(3n-1)(i-1)+j}=v_{i,j}$, $i=1,3m-1,\ j=1,3n-1,$ and first we number the components of the vectors at the nodes inside the first region, except for the nodes closest to its boundary, i. e., when $m+2\le i\le 2m-1,\ n+2\le j\le 2n-1,$ secondly, we number the components of the vectors at the nodes from the vicinity of the boundary of the first and second regions, i. e., when $m\le i\le 2m+1,\ n\le j\le 2n+1$ with the exception of the components of the vectors numbered earlier, i. e., when $m+2\le i\le 2m-1,\ n+2\le j\le 2n-1,$ and the third ones we will number the components of the vectors at the nodes inside the third region, i. e., when $1\le i\le 3m-1,\ 1\le j\le 3n-1$ with the exception of the components of the vectors numbered earlier, i. e., when $m\le i\le 2m+1,\ n\le j\le 2n+1.$ Note that the system of linear algebraic equations being solved, obtained by approximating the original and fictitious problems, can then be written in matrix form

$$\bar{u}\in R^N:\ B\bar{u}=\bar{f},\ \bar{f}\in R^N,\ N=(3m-1)(3n-1),$$

$$B=\begin{bmatrix}\Lambda_{11} & \Lambda_{12} & 0\\ 0 & \Lambda_{02} & \Lambda_{23}\\ 0 & \Lambda_{32} & \Lambda_{33}\end{bmatrix},\ \bar{u}=\begin{bmatrix}\bar{u}_1\\ \bar{0}\\ \bar{0}\end{bmatrix},\ \bar{f}=\begin{bmatrix}\bar{f}_1\\ \bar{0}\\ \bar{0}\end{bmatrix}.$$

A system for approximating the original problem

$$\bar{u}_1\in R^M:\ \Lambda_{11}\bar{u}_1=\bar{f}_1,\ \bar{f}_1\in R^M,\ M=(m-2)(n-2),\ m,n\ge 3,$$

The system obtained by approximating the corresponding fictitious problem

$$\begin{bmatrix}\Lambda_{02} & \Lambda_{23}\\ \Lambda_{32} & \Lambda_{33}\end{bmatrix}\begin{bmatrix}\bar{u}_2\\ \bar{u}_3\end{bmatrix}=\begin{bmatrix}\bar{f}_2\\ \bar{f}_3\end{bmatrix},\ \begin{bmatrix}\bar{f}_2\\ \bar{f}_3\end{bmatrix}=\begin{bmatrix}\bar{0}\\ \bar{0}\end{bmatrix},\ \begin{bmatrix}\bar{u}_2\\ \bar{u}_3\end{bmatrix}=\begin{bmatrix}\bar{0}\\ \bar{0}\end{bmatrix}.$$

When approximating a fictitious problem, we will use a change in the part of the domain of the coefficient a_{II} near part of the border $\partial\Omega_1\cap\partial\Omega_{\mathrm{II}}=\bar{S},\ S=\Gamma_{1,0}\cap\Gamma_{\mathrm{II},3}\ne\varnothing,$ so that when approximating the term with this coefficient, a diagonal matrix is obtained with the same values on the diagonal always equal a_{II}.

Algorithmic implementation of the iterative extension method for modeling plate deflection

We consider the method of iterative extensions for solving a system with a matrix B:

$$\bar{u}^k\in R^N: C(\bar{u}^k-\bar{u}^{k-1})=-\tau_{k-1}(B\bar{u}^{k-1}-\bar{f}),\ \tau_{k-1}=(\bar{r}^{k-1},\bar{\eta}^{k-1})/(\bar{\eta}^{k-1},\bar{\eta}^{k-1}),\ \tau_0=1,\ \bar{u}^0=\bar{0}\in R^N,$$

$$\bar r^{k-1} = B\bar u^{k-1} - \bar f,\ \bar w^{k-1} = C^{-1}\bar r^{k-1},\ \bar\eta^{k-1} = B\bar r^{k-1},\ k \in \mathrm{N}.$$

$$C = \Lambda_{\mathrm{I}} + \gamma\Lambda_{\mathrm{II}},\ \Lambda_{\mathrm{I}} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & 0 \\ \Lambda_{21} & \Lambda_{20} & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \Lambda_{\mathrm{II}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Lambda_{02} & \Lambda_{23} \\ 0 & \Lambda_{32} & \Lambda_{33} \end{bmatrix},\ \gamma = 1.$$

This iterative process is written as

$$\bar u^1 \in R^N : C\bar u^1 = \bar f;$$

$$\bar u^k \in R^N : C(\bar u^k - \bar u^{k-1}) = -\tau_{k-1}\Lambda_{\mathrm{II}}\bar u^{k-1},\ \tau_{k-1} = (\bar r^{k-1}, \bar\eta^{k-1}) \big/ (\bar\eta^{k-1}, \bar\eta^{k-1}),\ k \in \mathrm{N} \setminus \{1\}.$$

In an algorithm that implements an iterative process, the following is sequentially calculated:

I. The squared norm of the initial absolute error $\varepsilon_0 = (\bar f, \bar f)h^2$.

II. First approximation $\bar u^1 \in R^N : C\bar u^1 = \bar f$.

III. Discrepancy $\bar r^{k-1} = B\bar u^{k-1} - \bar f = \Lambda_{\mathrm{II}}\bar u^{k-1},\ k \in \mathrm{N} \setminus \{1\}$.

IV. The squared norm of the absolute error $\varepsilon_{k-1} = (\bar r^{k-1}, \bar r^{k-1})h^2,\ k \in \mathrm{N} \setminus \{1\}$.

V. Amendment $\bar w^{k-1} \in R^N : C\bar w^{k-1} = \bar r^{k-1},\ k \in \mathrm{N} \setminus \{1\}$.

VI. Equivalent discrepancy $\bar\eta^{k-1} = B\bar w^{k-1} = \Lambda_{\mathrm{II}}\bar w^{k-1},\ k \in \mathrm{N} \setminus \{1\}$.

VII. Iteration parameter $\tau_{k-1} = (\bar r^{k-1}, \bar\eta^{k-1}) \big/ (\bar\eta^{k-1}, \bar\eta^{k-1}),\ k \in \mathrm{N} \setminus \{1\}$.

VIII. Iterative approximation $\bar u^k = \bar u^{k-1} - \tau_{k-1}\bar w^{k-1},\ k \in \mathrm{N} \setminus \{1\}$.

IX. Iteration termination condition. $\varepsilon_{k-1} \le \varepsilon^2\varepsilon_0,\ k \in \mathrm{N} \setminus \{1\},\ \varepsilon = 0{,}001 \in (0;\, 1)$.

If the condition for stopping the iterations is not met, everything is repeated from point **III**.

An algorithm for calculating the first approximation and correction in an algorithm implementing the iterative extension method

Note that the system of linear algebraic equations being solved, obtained at each step of the previously applied iterative process, is written in matrix form

$$\bar v \in R^N :\ C\bar v = \bar g,\ \bar g \in R^N,\ N = (3m-1)(3n-1).$$

To solve this problem, an iterative process of the form is used

$$\bar v^l \in \mathbb{R}^N : A^2(\bar v^l - \bar v^{l-1}) = -\tau_{l-1}(C\bar v^{l-1} - \bar g),\ \ \tau_{l-1} > 0,\ \bar u^0 = \bar 0 \in \mathbb{R}^N,\ l \in \mathbb{N}.$$

Here is the matrix C coincides with the matrix up to a permutation of rows $(A - \kappa E)^2 + aE,\ \kappa = \sqrt{a},$ where E identity matrix of dimension $N \times N$, matrix $A - \kappa E$ determined from the difference scheme:

$$\frac{4v_{i,j} - v_{i-1,j} - v_{i+1j} - v_{i,j-1} - v_{i,j+1}}{h^2},\ 1 \le i \le 3m-1,\ 1 \le j \le 3n-1,$$

$$v_{i,0}^k = v_{i,1}^k,\ i = 1,2,\ldots,3m-1,\ v_{0,j}^k = v_{1,j}^k,\ \ j = 1,2,\ldots,3n-1,$$

$$v_{i,3n}^k = 0,\ i = 1,2,\ldots,3m-1,\ v_{3m,j}^k = 0,\ \ j = 1,2,\ldots,3n-1,$$

$$a = a_1,\ \text{if}\ \ m+2 \le i \le 2m-1,\ \ n+2 \le j \le 2n-1,\ \text{then}\ \ a = a_{\mathrm{II}}.$$

To select the iteration parameter, the well-known method of minimum corrections [10] is used. The algorithm calculates:

I. Initial approximation $\bar v^0 = \bar 0 \in \mathbb{R}^N$.

II. Discrepancy $\bar r^{l-1} : \bar r^{l-1} = C\bar v^{l-1} - \bar g,\ l \in \mathbb{N}.$ More

$$\bar t^{l-1} : \bar t^{l-1} = (A - \kappa E)\bar v^{l-1},\ l \in \mathbb{N},\ \ \bar r^{l-1} : \bar r^{l-1} = (A - \kappa E)\bar t^{l-1} + a\bar v^{l-1} - \bar g,\ l \in \mathbb{N}.$$

III. Amendment $\bar w^{l-1} \in \mathbb{R}^N : A^2\bar w^{l-1} = \bar r^{l-1},\ l \in \mathbb{N}$.

IV. Square of the error norm $E_{l-1} = \left\| \bar\psi^{l-1} \right\|^2_{C'A^{-2}C} = \left\langle \bar r^{l-1}, \bar w^{l-1} \right\rangle,\ l \in \mathbb{N}$.

V. Iteration termination condition $E_{l-1} \le E_0\varepsilon^2$, $\varepsilon \in (0;1)$, $l \in \mathbb{N}$.

VI. Equivalent discrepancy $\overline{\eta}^{l-1} : \overline{\eta}^{l-1} = C\overline{w}^{l-1}$, $l \in \mathbb{N}$. More

$$\overline{p}^{l-1} : \overline{p}^{l-1} = (A-\kappa E)\overline{w}^{l-1},\ l \in \mathbb{N},\ \ \overline{\eta}^{l-1} : \overline{\eta}^{l-1} = (A-\kappa E)\overline{p}^{l-1} + a\overline{w}^{l-1},\ l \in \mathbb{N}.$$

VII. Equivalent correction $\overline{\xi}^{l-1} \in \mathbb{R}^N : A^2\overline{\xi}^{k-1} = \overline{\eta}^{l-1}$, $l \in \mathbb{N}$.

VIII. Iteration parameter

$$\tau_{l-1} = \frac{\left\langle \overline{w}^{l-1}, C\overline{w}^{l-1} \right\rangle}{\left\langle C\overline{w}^{l-1}, A^{-2}C\overline{w}^{l-1} \right\rangle} = \frac{\left\langle \overline{w}^{l-1}, \overline{\eta}^{l-1} \right\rangle}{\left\langle \overline{\eta}^{l-1}, \overline{\xi}^{l-1} \right\rangle},\ l \in \mathbb{N}.$$

IX. Iterative solution $\overline{v}^l = \overline{v}^{l-1} - \tau_{l-1}\overline{w}^{l-1}$, $l \in \mathbb{N}$.

Note that the relative error specified in the iterative process termination condition is:

$$\varepsilon = 0{,}001 \in (0;1).$$

In the iterative process, a problem of the form:

$$\overline{v} \in \mathbb{R}^N : A^2\overline{v} = \overline{g},\ \overline{g} \in \mathbb{R}^N,$$

which is written as two problems:

$$\overline{q} \in \mathbb{R}^N : A\overline{q} = \overline{g},\ \overline{g} \in \mathbb{R}^N,\ \ \overline{v} \in \mathbb{R}^N : A\overline{v} = \overline{q},\ \overline{q} \in \mathbb{R}^N.$$

When solving problems of the previous type, the well-known marching method can be used [5–8].

Calculation of the deflection of a shipbuilding plate

The following data were considered in the calculations:

$l_1 = 1\text{м}$, $b_1 = 1\text{м}$, $H = 0{,}03\text{м}$, $K_1 = 0\,\text{Па/м}$, $E_1 = 2\cdot 10^{11}\,\text{Па}$, $\sigma_1 = 0{,}3\,\text{м/м}$, $P_1 = 80\,000\,\text{Па}$, $\varepsilon = 0{,}001$,

where $D_1 = 494\,505{,}5\,\text{Па}\times\text{м}^3$, $a_1 = 0\,\text{м}^{-4}$, $f_1 = 0{,}162\,\text{м}^{-3}$. A table of the number of iterations in calculations on a computer with a given number of nodes was obtained.

Number of iterations depending on the number of nodes in the directions of the axes

m, n	41	71	101	131	161
k	7	5	5	4	4

The maximum approximate value of the slab deflection, i. e. the maximum value of the modulus of the last iterative solution on the finest grid $u_{i,j}^k$:

$$M = \max_{\substack{m+2\le i\le 2m-1\\ n+2\le j\le 2n-1}} \left|u_{i,j}^k\right| = 0{,}000\,205\,299\ \text{м}.$$

Conclusion

A detailed algorithm has been developed for the numerical modeling of the deflection of a rectangular plate under a transverse load, where the plate is located on an elastic foundation and rigidly fixed at its edges. The proposed algorithm is asymptotically optimal in terms of computational effort, as it is based on the iterative expansion method. The algorithm's operation is successfully demonstrated in the computer calculation of a ship's plate.

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ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ПРОГИБА ПРЯМОУГОЛЬНОЙ ПЛАСТИНЫ НА УПРУГОМ ОСНОВАНИИ ПРИ ЕЕ ЖЕСТКОМ ЗАКРЕПЛЕНИИ ПО КРАЯМ

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Аннотация. Приведен алгоритм численного моделирования прогиба прямоугольной пластины на упругом основании при ее жестком закреплении по краям. Предложенный алгоритм асимптотически оптимальный по вычислительным затратам и основывается на методе итерационных расширений и предполагает использование маршевых методов. Асимптотическая оптимальность алгоритма экспериментально подтверждена при расчетах на ЭВМ.

Ключевые слова: численное моделирование; прогиб прямоугольной пластины.

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