

THE BARENBLATT–ZHELTOV–KOCHINA EQUATION WITH BOUNDARY NEUMANN CONDITION AND MULTIPOINT INITIAL-FINAL VALUE CONDITION

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The article is devoted to the study of the unique solvability of the Barenblatt–Zhel'tov–Kochina equation, equipped with the Neumann boundary condition and a multipoint initial-final value condition. This equation is degenerate or, in other words, it belongs to the Sobolev type equations. To study this equation, the authors used the methods of the theory of degenerate operator semigroups, created by Prof. G.A. Sviridyuk, and further developed by him and his students. We would also like to note that the equation under study is supplied with a multipoint initial-final value condition, which is not just a generalization of the Cauchy problem for the Sobolev type equations. This condition makes it possible to avoid checking the consistency of the initial data when finding a solution.

Keywords: Barenblatt–Zhel'tov–Kochina equation; Neumann condition; multipoint initial-final value condition; unique solvability.

Introduction

Let \mathfrak{A} and \mathfrak{F} be Banach spaces; operator $L \in \mathcal{L}(\mathfrak{A}; \mathfrak{F})$ (i.e. linear and continuous), and operator $M \in Cl(\mathfrak{A}; \mathfrak{F})$ (i.e. linear, closed and densely defined). Following [1], we introduce into consideration L -resolvent set $\rho^L(M) = \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathfrak{F}; \mathfrak{A})\}$ and L -spectrum $\sigma^L(M) = \mathbb{C} \setminus \rho^L(M)$ of operator M . The following statements are true.

Theorem 1. [1] *Let operator M be (L, p) -bounded, $p \in \{0\} \cup \mathbb{N}$. Then there exist such projectors $P: \mathfrak{A} \rightarrow \mathfrak{A}$ and $Q: \mathfrak{F} \rightarrow \mathfrak{F}$, that operators $L \in \mathcal{L}(\ker P; \ker Q) \cap \mathcal{L}(\operatorname{im} P; \operatorname{im} Q)$ and $M \in Cl(\ker P; \ker Q) \cap Cl(\operatorname{im} P; \operatorname{im} Q)$.*

Introduce the following condition

$$\left\{ \begin{array}{l} \sigma^L(M) = \bigcup_{j=0}^n \sigma_j^L(M), n \in \mathbb{N}, \text{ what is more } \sigma_j^L(M) \neq \emptyset, \text{ there exists} \\ \text{a closed contour } \gamma_j \subset \mathbb{C}, \text{ bounding a domain } D_j \supset \sigma_j^L(M), \\ \text{such, that } \bar{D}_j \cap \sigma_0^L(M) = \emptyset, \bar{D}_k \cap \bar{D}_l = \emptyset \quad \forall j, k, l = 1, \dots, n, k \neq l. \end{array} \right. \quad (\text{A})$$

Theorem 2. [2] *Let operator M be (L, p) -bounded, $p \in \{0\} \cup \mathbb{N}$, and condition (A) is fulfilled. Then there exist projectors $P_j \in \mathcal{L}(\mathfrak{A})$ and $Q_j \in \mathcal{L}(\mathfrak{F})$, $j = 1, \dots, n$, having the form*

$$P_j = \frac{1}{2\pi i} \int_{\gamma} (\mu L - M)^{-1} L d\mu, \quad Q_j = \frac{1}{2\pi i} \int_{\gamma} L (\mu L - M)^{-1} d\mu, \quad j = 1, \dots, n.$$

Moreover, one more statement is true.

Corollary 1. *Let conditions of theorems 1 and 2 are satisfied. Then $P P_j = P_j P = P_j$, $j = 1, \dots, n$, $P_k P_l = P_l P_k = \mathbb{O}$, $k, l = 1, \dots, n$, $k \neq l$; $Q Q_j = Q_j Q = Q_j$, $j = 1, \dots, n$, $Q_k Q_l = Q_l Q_k = \mathbb{O}$, $k, l = 1, \dots, n$, $k \neq l$.*

Put

$$P_0 = P - \sum_{k=1}^n P_k, \quad Q_0 = Q - \sum_{k=1}^n Q_k,$$

due to corollary 1 operators $P_0 \in \mathcal{L}(\mathfrak{A})$, $Q_0 \in \mathcal{L}(\mathfrak{F})$ are projectors.

Thus, let condition (A) is fulfilled, fix $\tau_j \in \mathbb{R}$ ($\tau_j < \tau_{j+1}$), vectors $u_j \in \mathfrak{A}$ for $j=0, \dots, n$, and consider multipoint initial-final value condition [2] (see more [3])

$$P_j(u(\tau_j) - u_j) = 0, \quad j=0, \dots, n, \quad (1)$$

for linear Sobolev type equation

$$Lu = Mu + f, \quad (2)$$

where vector-function $f \in C^\infty(\mathbb{R}; \mathfrak{F})$ will be defined below.

A vector-function $u \in C^\infty(\mathbb{R}; \mathfrak{A})$, satisfying equation (2), is called a *solution of equation (2)*. Solution $u = u(t)$, $t \in \mathbb{R}$, of equation (2), satisfying conditions (1) is called *the solution of multipoint initial-final value problem (1), (2)*.

In this paper we present the results of the Sobolev type equations theory with (L, p) -bounded operator M [1] and the unique solvability of problem (1), (2) [2]. Then the abstract results will be applied to the study of the solvability of Barenblatt–Zhel'tov–Kochina equation

$$u_t - \chi \Delta u_t = \nu \Delta u + f, \quad (3)$$

defined in cylinder $\Omega \times \mathbb{R}^m$ with boundary conditions

$$\frac{\partial u}{\partial n} u(x, t) = 0(x), \quad (x, t) \in \partial \Omega \times \mathbb{R}, \quad (4)$$

and with multipoint initial-final value condition of the form (1). Here $\Omega \subset \mathbb{R}^m$ is a bounded domain with the boundary of the class C^∞ , and $n = n(x)$, $x \in \partial \Omega$, is the unit normal external to domain Ω . Thus, the subject of the paper is divided into two parts. In the first part the information on the solvability of problem (1), (2) is given, and in the second part problem (1), (3), (4) is considered.

Note, that the Neumann conditions are a special case of the “flow balance” condition for Sobolev type equations considered on a connected oriented graph, that is in the one-dimensional case. This theory is currently being actively developed, the first studies were conducted in [4]. Sobolev type equations with Cauchy–Neumann conditions in a bounded domain were studied in [5], but studies for the case of replacing the Cauchy condition by a multipoint initial-final value condition for such a problem are considered here for the first time.

We also note, that equation (3) models the dynamics of the pressure of a fluid, filtered in a fractured-porous medium [6]. Here χ is a real parameter characterizing the medium, ν is the piezoconductivity coefficient of the fractured rock, with $\chi \in \mathbb{R}$, $\nu \in \mathbb{R}_+$; function $f = f(x)$ plays the role of an external load. In addition, equation (3) describes the flow of second-order liquids [7], the heat conduction process with “two temperatures” [8], the moisture transfer process in the soil [9].

1. Abstract problem

Let \mathfrak{A} and \mathfrak{F} be Banach spaces, operators $L \in \mathcal{L}(\mathfrak{A}; \mathfrak{F})$ (i.e. linear and continuous) and $M \in Cl(\mathfrak{A}; \mathfrak{F})$ (i.e. linear, closed and densely defined). Suppose, in addition, operator M is (L, σ) -bounded (for terminology and results see [1]), then there exist degenerate analytical groups of resolving operators

$$U^t = \frac{1}{2\pi i} \int_\gamma R_\mu^L(M) e^{\mu t} d\mu \quad u \quad F^t = \frac{1}{2\pi i} \int_\gamma L_\mu^L(M) e^{\mu t} d\mu,$$

defined on spaces \mathfrak{A} and \mathfrak{F} respectively, moreover $U^0 \equiv P$, $F^0 \equiv Q$ are projectors. Here γ is a contour, bounding a domain D , containing L -spectrum $\sigma^L(M)$ of operator M ; $R_\mu^L(M) = (\mu L - M)^{-1} L$ is a *right*, and $L_\mu^L(M) = L(\mu L - M)^{-1}$ is a *left L-resolvent* of operator M . For degenerate analytical group the concepts of *kernel* $\ker U^\cdot = \ker P = \ker U^t$, for all $t \in \mathbb{R}$ and *image* $\text{im} U^\cdot = \text{im} P = \text{im} U^t$ for all $t \in \mathbb{R}$ are correct. Denote by $\mathfrak{A}^0 = \ker U^\cdot$, $\mathfrak{A}^1 = \text{im} U^\cdot$, and $\mathfrak{A}^0 = \ker F^\cdot$, $\mathfrak{F}^1 = \text{im} F^\cdot$, then $\mathfrak{A}^0 \oplus \mathfrak{A}^1 = \mathfrak{A}$ and $\mathfrak{F}^0 \oplus \mathfrak{F}^1 = \mathfrak{F}$. Also denote by $L_k(M_k)$ the restriction of operator $L(M)$ on \mathfrak{F}^k ($\text{dom} M \cap \mathfrak{A}^k$), $k=0, 1$.

Theorem 3. [1] (Splitting theorem). *Let operator M be (L, p) -bounded. Then*

- (i) operators $L_k \in \mathcal{L}(\mathfrak{A}^k; \mathfrak{F}^k)$, $k = 0, 1$;
- (ii) operators $M_0 \in Cl(\mathfrak{A}^0; \mathfrak{F}^0)$, $M_1 \in \mathcal{L}(\mathfrak{A}^1; \mathfrak{F}^1)$;
- (iii) there exist operators $L_1^{-1} \in \mathcal{L}(\mathfrak{F}^1; \mathfrak{A}^1)$ and $M_0^{-1} \in \mathcal{L}(\mathfrak{F}^0; \mathfrak{A}^0)$.

Put $H = M_0^{-1}L_0 \in \mathcal{L}(\mathfrak{A}^0)$, $S = L_1^{-1}M_1 \in \mathcal{L}(\mathfrak{A}^1)$. It is true

Corollary 2. [1] *Let operator M be (L, σ) -bounded. Then for all $\mu \in \mathbb{C} \setminus \bar{D}$*

$$(\mu L - M)^{-1} = -\sum_{k=0}^{\infty} \mu^k H^k M_0^{-1} (I - Q) + \sum_{k=1}^{\infty} \mu^{-k} S^{k-1} L_1^{-1} Q.$$

An operator M is called (L, p) -bounded, $p \in \{0\} \cup \mathbb{N}$, if $H^p \neq \mathbb{O}$, and $H^{p+1} = \mathbb{O}$. Let condition (A) is satisfied. Then takes place:

Theorem 4. [2] *Let operator M be (L, p) -bounded, $p \in \{0\} \cup \mathbb{N}$, and condition (A) is fulfilled.*

Then

- (i) there exist degenerate analytical groups

$$U_j^t = \frac{1}{2\pi i} \int_{\gamma} R_{\mu}^L(M) e^{\mu t} d\mu, j = \overline{1, n}.$$

- (ii) $U^t U_j^s = U_j^s U^t = U_j^{s+t}$ for all $s, t \in \mathbb{R}$, $j = \overline{1, n}$;

- (iii) $U_k^t U_l^s = U_l^s U_k^t = \mathbb{O}$ for all $s, t \in \mathbb{R}$, $k, l = \overline{1, n}$, $k \neq l$.

Put $U_0^t = U^t - \sum_{k=1}^n U_k^t$, $t \in \mathbb{R}$.

Remark 1. Units $P_j \equiv U_j^0$, $j = \overline{0, n}$, (constructed by virtue of condition (A)) of degenerate analytical groups $\{U_j^t : t \in \mathbb{R}\}$, $j = \overline{0, n}$, are projectors by corollary (1). We call the operators P_j , Q_j , $j = \overline{0, n}$, relatively spectral projectors.

Consider subspaces $\mathfrak{A}^{1j} = \text{im } P_j$, $\mathfrak{F}^{1j} = \text{im } Q_j$, $j = \overline{0, n}$. By construction

$$\mathfrak{A}^1 = \bigoplus_{j=0}^n \mathfrak{A}^{1j} \quad \text{and} \quad \mathfrak{F}^1 = \bigoplus_{j=0}^n \mathfrak{F}^{1j}.$$

Denote by L_{1j} the restriction of operator L on \mathfrak{A}^{1j} , $j = \overline{0, n}$, and by M_{1j} denote the restriction of operator M on $\text{dom } M \cap \mathfrak{A}^{1j}$, $j = \overline{0, n}$. Since, as it easy to show, that $P_j \varphi \in \text{dom } M$, if $\varphi \in \text{dom } M$, then the domain $\text{dom } M_{1j} = \text{dom } M \cap \mathfrak{A}^{1j}$ is dense in \mathfrak{A}^{1j} , $j = \overline{0, n}$.

Theorem 5. [2] (Generalized spectral theorem). *Let operators $L \in \mathcal{L}(\mathfrak{A}; \mathfrak{F})$ and $M \in Cl(\mathfrak{A}; \mathfrak{F})$, operator M is (L, p) -bounded, $p \in \{0\} \cup \mathbb{N}$, and condition (A) is fulfilled. Then*

- (i) operators $L_{1j} \in \mathcal{L}(\mathfrak{A}^{1j}; \mathfrak{F}^{1j})$, $M_{1j} \in \mathcal{L}(\mathfrak{A}^{1j}; \mathfrak{F}^{1j})$, $j = \overline{0, n}$;
- (ii) there exist operators $L_{1j}^{-1} \in \mathcal{L}(\mathfrak{F}^{1j}; \mathfrak{A}^{1j})$, $j = \overline{0, n}$.

Theorem 6. [2] *Let operator M is (L, p) -bounded, $p \in \{0\} \cup \mathbb{N}$, moreover condition (A) is fulfilled. Then for all $f \in C^{\infty}(\mathbb{R}; \mathfrak{F})$, $u_j \in \mathfrak{A}$, $j = \overline{0, n}$, there exists the unique solution of problem (1), (2), having the form*

$$u(t) = -\sum_{q=0}^p H^q M_0^{-1} (\mathbb{I} - Q) f^{(q)}(t) + \sum_{j=0}^n U_j^{t-\tau_j} u_j + \sum_{j=0}^n \int_{\tau_j}^t U_j^{t-s} L_{1j}^{-1} Q_j f(s) ds. \quad (5)$$

2. Concrete interpretation

Reduce problem (3), (4) to equation (2). For this we set

$$\mathfrak{A} = \left(u \in W_2^{m+2} : \frac{\partial u}{\partial n} = 0 \text{ on } \partial\Omega \right), \mathfrak{F} = W^m, m \in \mathbb{N}.$$

All functional spaces are defined on the domain Ω . Let us set the operators $L = \mathbb{I} - \chi\Delta$, $M = \nu\Delta$, moreover $L, M \in \mathcal{L}(\mathfrak{A}; \mathfrak{F})$ for all $\chi \in \mathbb{R} \setminus \{0\}$, $\nu \in \mathbb{R}$, and operator L is Fredholm (i.e. $\text{ind } L = 0$).

Denote by $\{\lambda_k\}$ the set of eigenvalues of homogeneous Neumann problem for the Laplace operator Δ in the domain Ω , numbered in the order of non-increasing with allowance for their multiplicity, and by the $\{\varphi_k\}$ denote the set of orthonormalized (in the sense of the space L_2) corresponding eigenvectors. Note, that the first eigenvalue of the homogeneous Neumann problem for the Laplace operator in domain Ω is zero, and the corresponding eigenfunction is constant.

Lemma 1. [5] For all $\chi, \nu \in \mathbb{R} \setminus \{0\}$ operator M is $(L, 0)$ -bounded.

Note, that

$$\ker L = \begin{cases} \{0\}, & \text{if } \chi^{-1} \notin \{\lambda_k\}, \\ \text{span}\{\varphi_l : \chi^{-1} = \lambda_l\}. & \end{cases}$$

By theorem 1 we construct a projector

$$P = \begin{cases} \mathbb{I}, & \text{if } \chi^{-1} \notin \{\lambda_l\}, \\ \mathbb{I} - \sum_{l: \chi^{-1} = \lambda_l} \langle \cdot, \varphi_l \rangle \varphi_l, & \text{if } \chi^{-1} \in \{\lambda_l\}, \end{cases}$$

where $\langle \cdot, \cdot \rangle$ is the scalar product in L^2 . Projector Q has the same form, but it is defined on the space \mathfrak{F} . The relative spectrum $\sigma^L(M)$ of operator M has the form

$$\sigma^L(M) = \left(\mu_k = \frac{\nu\lambda_k}{1 - \chi\lambda_k}, k \in \mathbb{N} \right).$$

Choose such the parts $\sigma_j^L(M)$, $j = \overline{0, n}$, of the relative spectrum of operator M , that condition (A) is satisfied (it is clear that this can done in more then one way). Build the projectors

$$P_j = \sum_{k: \mu_k \in \sigma_j^L(M)} \langle \cdot, \varphi_k \rangle \varphi_k, j = \overline{0, n}.$$

Take $\tau_j \in \mathbb{R}$ ($\tau_j < \tau_{j+1}$), $u_j \in \mathfrak{A}$, $j = \overline{0, n}$, $f \in C^\infty(\mathbb{R}; \mathfrak{F})$ and for problem (3), (4) the multipoint initial-final value condition is given

$$\sum_{k: \mu_k \in \sigma_j^L(M)} \langle u(\tau_j, x) - u^j(x), \varphi_k \rangle \varphi_k = 0, j = \overline{0, n}. \quad (6)$$

By lemma 1 and theorem 6 it follows

Theorem 7. Let condition (A) is fulfilled. For all $\chi \in \mathbb{R}$, $\nu \in \mathbb{R} \setminus \{0\}$, $f \in C^\infty(\mathbb{R}; \mathfrak{F})$, $u_j \in \mathfrak{A}$, $j = \overline{0, n}$, equation (3) with conditions (4), (6) has the unique solution $u \in C^\infty(\mathbb{R}; \mathfrak{A})$, which has the form

$$u(t) = (Q - \mathbb{I})f(t) + \sum_{j=0}^n \sum_{\mu_k \in \sigma_j^L(M)} e^{\mu_k(t-\tau_j)} \langle u_j, \varphi_k \rangle \varphi_k + \sum_{j=0}^n \sum_{\mu_k \in \sigma_j^L(M)} \tau_j \int_0^t e^{\mu_k(t-s)} \langle f(s), \varphi_k \rangle \varphi_k ds.$$

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УРАВНЕНИЕ БАРЕНБЛАТТА–ЖЕЛТОВА–КОЧИНОЙ С ГРАНИЧНЫМ УСЛОВИЕМ НЕЙМАНА И МНОГОТОЧЕЧНЫМ НАЧАЛЬНО-КОНЕЧНЫМ УСЛОВИЕМ

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Посвящена изучению однозначной разрешимости уравнения Баренблатта–Желтова–Кочиной, снабженного краевым условием Неймана и многоточечным начально-конечным условием. Отметим, что уравнение Баренблатта–Желтова–Кочиной моделирует динамику давления жидкости, фильтрующейся в трещиновато-пористой среде. Кроме того, оно описывает течение жидкостей второго порядка, процесс теплопроводности с «двумя температурами», процесс влагопереноса в почве. Данное уравнение является вырожденным или, другими словами, оно принадлежит к уравнениям соболевского типа. Для исследования изучаемого уравнения авторы воспользовались методами теории вырожденных полугрупп операторов, разработанной проф. Г.А. Свиридюком, и развитой его учениками. Отметим также, что исследуемое уравнение снабжено многоточечным начально-конечным условием, которое является не просто обобщением задачи Коши для уравнений соболевского типа. Указанное условие дает возможность избежать проверки согласования начальных данных при нахождении решения.

Ключевые слова: уравнение Баренблатта–Желтова–Кочиной; условие Неймана; многоточечное начально-конечное условие; однозначная разрешимость.

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